MTH 150 **Name: ________________________**
Prerequisite Preparation

Initial Skills Packet

Welcome to Calculus! Calculus is a lively, dynamic and interesting subject that touches nearly every discipline. Calculus influences the technical (engineering, biology, chemistry, physics, medicine, computer science, etc.) as well as the less technical (business, education, psychology, etc.) Unfortunately, learning Calculus in the traditional sense is nearly impossible without a thorough grasp of both Algebra and Trigonometry. What follows are some sample problems that you should be able to do BEFORE entering Calculus. It is important to understand that these problems do not serve as the *only* problems that must be mastered. Rather, it should give you an idea of the types of things you should have internalized at the completion of a solid Algebra and Trigonometry sequence.

Completing this Sample Packet with 60-70% accuracy is important (the other 30-40% will probably "come back" as you learn Calculus). *Remember that this is just a sample*. The actual exam you take will be much shorter in length but will test the same basic core material. No calculators are allowed on the exam.

ARITHMETIC/NUMERICAL REASONING

1. Simplify $\frac{1}{2}+\frac{2}{3}-\frac{4}{5}$.

2. Is $\frac{8\times10^{-3}}{2\times10^{-5}}$ a large or small number? (Circle one) **LARGE** **SMALL**

3. What is the meaning of the statement $\log_3 81 = 4$?

4. Why is "division by zero" not allowed?

5. Fill in the blanks below with the narrowest estimate possible. Use whole numbers (positive or negative) that are one unit apart.

   Example: ______ $< \frac{4}{5} <$ ______ *Solution*: $0 < \frac{4}{5} < 1$

   (a) ______ $< \cos\left(\frac{7\pi}{10}\right) <$ ______

   (b) ______ $< \log_4 30 <$ ______

(c) ______ < e^{-1} < ______

6. The mathematical constants π and e play important roles in mathematics. What are their values (approximately)? In what context(s) do they appear?

7. The number 7^{-2} is what kind of number (positive or negative)? Explain.

ELEMENTARY ALGEBRA

8. Express $\frac{1}{x+1}+\frac{2x}{x+3}$ as a single fraction.

9. Find the exact solution(s) to $2x^2-8x+7=6$.

10. Find the equation of the line passing through $(1,\ 2)$ and $(-4,\ 5)$.

11. Write $-2x^{-5}$ without negative exponents.

12. Factor the following expressions. If prime, so state.

(a) $3x^2-3x-x+1$ (a) ____________________

(b) x^3-x^2+x+1 (b) ____________________

(c) $2x^2-3x-4$ (c) ____________________

(d) x^2-25 (d) ____________________

(e) x^2+25 (e) ____________________

(f) $3x^4-5x^2+2$ (f) ____________________

(g) $8y^3-27$ (g) ____________________

ADVANCED ALGEBRA & FUNCTIONS

13. Given that $f(x)=-2x^2+5x-6$ and $g(x)=x-4$, find and/or simplify the following:

(a) $f(-1)$ (a) ____________

(b) $g(10)$ (b) ____________

(c) $f(x+h)$ (c) ____________

(d) $g(a-b)$ (d) ____________

(e) $f(g(x))$ (e) ____________

(f) $(g \circ f)(x)$ (f) ____________

14. Simplify $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$ so that no radicals appear in the numerator of the fraction.

15. Simplify the complex rational expression $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$.

16. Follow the logic in this argument: Consider the statement $a=b\neq 0$. Multiply both sides of the equation $a=b$ by a to get $a^2=ab$. Next, subtract b^2 from both sides to get $a^2-b^2=ab-b^2$. Continuing, we have $(a+b)(a-b)=b(a-b)$. Now divide both sides by $a-b$ to get $a+b=b$. Since $a=b$ by assumption, we have $b+b=b$ or $2b=b$. Since $b\neq 0$, we have $2=1$. Where is the illogical step?

17. Consider the function $y=f(x)$. Generally speaking, if $f(5)=-2$, what is the value of $f^{-1}(-2)$? Explain.

18. For the functions below, find the domain, range, and a formula for its inverse function.

(a) $f(x)=(x-4)^2+7$
(b) $g(x)=\sqrt{2x-5}$

TRIGONOMETRY

19. Evaluate (a) $\sin\left(\dfrac{\pi}{4}\right)$ (a) ______________

(b) $\tan\left(\dfrac{\pi}{6}\right)$ (b) ______________

(c) $\cos\left(-\frac{\pi}{3}\right)$ (c) ______________

20. Evaluate (a) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ (a) ______________

(b) $\arcsin\left(\sin\frac{2\pi}{3}\right)$ (b) ______________

(c) $\sin\left(\arcsin\frac{2\pi}{3}\right)$ (c) ______________

21. Explain why $\sin(-x) = -\sin x$ whereas $\cos(-x) = \cos x$.

22. For the functions below, find the domain, range, and a formula for its inverse function.

(a) $k(x) = \cos(2x)$
(b) $j(x) = \tan x$

MISCELLANEOUS

23. Identify as TRUE or FALSE. Assume all expressions denote real numbers. Note: For those that are TRUE, you should reflect on why this is so; for those that are FALSE, you should provide a counterexample.

(a) ____________ $\sqrt{x^2 + y^2} = x + y$

(b) ____________ $\sin^2\theta + \cos^2\theta = 1$ for any angle θ.

(c) ____________ $\ln(A - B) = \frac{\ln A}{\ln B}$

(d) ____________ $\ln A^p = p \ln A$

(e) ____________ $\sin(\theta + 100\pi) = \sin\theta$ for any angle θ.

(f) ____________ $\sqrt{ab} = \sqrt{a}\sqrt{b}$

(g) ______________ $\arcsin(\sin x) = x$ for any number x.

(h) ______________ Given any number x, $x^2 \geq x$.

(i) ______________ $e^{A+B} = e^A e^B$

(j) ______________ $(x^m)^n = x^{m^n}$

(k) ______________ $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

(l) ______________ $x^{1/2} = \sqrt{x}$

(m) ______________ A function may have two *x*-intercepts.

(n) ______________ A function may have two *y*-intercepts.

(o) ______________ For any function f, $f(3-2) = f(3) - f(2)$