MTH 150 **Name: ______________________**
Initial Skills Exam (Sample) **Date: ______________________**
Key

<u>Directions</u>: **Answer each question to the best of your ability. Be sure to provide explanations where requested (use the space provided). No calculators please.**

For items 1-3, show all work.

1. TRUE or $\boxed{\text{FALSE}}$: For any numbers *a* and *b*, $\sqrt{a^2+b^2}=a+b$. Explain your answer.

   This is false in general. It may work for some choices of *a* and *b*. For example, if $a=5$ and $b=0$ then it is true that $\sqrt{5^2+0^2}=5+0$. However, if $a=2$ and $b=3$ then $\sqrt{2^2+3^2}=2+3$ leads to $\sqrt{13}=5$, which is a false statement.

2. TRUE or $\boxed{\text{FALSE}}$: If *x* is any number, then $x^2 \ge x$. Explain your answer.

   This is false in general. Similar to item # 1, this will be true for many choices of *x*. For example, if $x=4$ then it is indeed true that $4^2 \ge 4$. Also, equality holds if $x=0$ or $x=1$. If one chooses a common fractional value such as $x=\frac{1}{2}$, then the statement reads $\left(\frac{1}{2}\right)^2 \ge \frac{1}{2}$ or $\frac{1}{4} \ge \frac{1}{2}$, a false statement.

3. Factor $2x^4-6x^3-8x^2+24x$ <u>completely</u>. Show all work.

   There are numerous ways to approach this. Here is a straightforward method:

$$\begin{aligned} 2x^4-6x^3-8x^2+24x &= 2x\left(x^3-3x^2-4x+12\right) \\ &= 2x\left(x^2(x-3)-4(x-3)\right) \\ &= 2x\left((x-3)\left(x^2-4\right)\right) \\ &= 2x(x-3)(x+2)(x-2). \quad \leftarrow \text{Answer} \end{aligned}$$

Items 4-10 are multiple choice. Circle your answer.

4. Find $\sin\left(\frac{2\pi}{3}\right)$.

   $\boxed{\text{(a)}}$ $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$

   (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

There are many ways to approach this (recall/memorization, reference angles, special triangles, graph of the sine wave, unit circle, etc.). Using the fact that π radians is equivalent to 180°, $\frac{2\pi}{3}$ radians is equivalent to 120°. Then, recall from the symmetry of the sine wave (or the unit circle) that $\sin 120^{\circ} = \sin 60^{\circ}$. Since $\sin 60^{\circ} = \frac{\sqrt{3}}{2}$, the answer is (a).

5. Find the solution(s) to $\sin^2\theta = \frac{1}{2}$ on the interval $[0, 2\pi]$.

(a) $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ (b) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

[(c)] $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, and $\frac{7\pi}{4}$ (d) $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, and $\frac{11\pi}{6}$

The statement $\sin^2\theta = \frac{1}{2}$ implies that $\sin\theta = \pm\sqrt{\frac{1}{2}}$ after taking a square root. Rationalizing the right-hand side reveals the more familiar form $\pm\frac{\sqrt{2}}{2}$ so we have $\sin\theta = \pm\frac{\sqrt{2}}{2}$. Then use your knowledge of key sine values and periodicity to conclude that $\theta = \frac{\pi}{4}$ and $\theta = \pi - \frac{\pi}{4}$ for $\sin\theta = \frac{\sqrt{2}}{2}$ and $\theta = \pi + \frac{\pi}{4}$ and $\theta = 2\pi - \frac{\pi}{4}$ for $\sin\theta = -\frac{\sqrt{2}}{2}$. Thus, the theta values are $\theta = \frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, and $\frac{7\pi}{4}$. The answer is (c).

6. Evaluate $\arctan\frac{\sqrt{3}}{3}$.

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$

[(c)] $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$

The expression $\arctan\frac{\sqrt{3}}{3} = ?$ is equivalent to asking $\tan(?) = \frac{\sqrt{3}}{3}$. Basic recall says 30° or $\frac{\pi}{6}$. The answer is (c).

7. Find the exact solutions to $x^2 + 4x = 7$.

(a) $x = 3$ and $x = 7$ | [(b)] $x = -2 + \sqrt{11}$ and $x = -2 - \sqrt{11}$

(c) $x = 3$ and $x = -\frac{1}{2}$ | (d) $x = -2 + i\sqrt{3}$ and $x = -2 - i\sqrt{3}$

Completing the square or the quadratic formula will lead to the solution. $x^2 + 4x = 7$ leads to $x^2 + 4x + 4 = 7 + 4$ or $(x+2)^2 = 11$ so $x + 2 = \pm\sqrt{11}$ and finally $x = -2 \pm \sqrt{11}$. Choose (b).

8. Given that $f(x) = x^3 + 5$, find $f^{-1}(x)$.

(a) $f^{-1}(x) = \sqrt[3]{x} - \sqrt[3]{5}$ | (b) $f^{-1}(x) = \sqrt[3]{x} - 5$

(c) $f^{-1}(x) = \dfrac{1}{x^3 + 5}$ | [(d)] $f^{-1}(x) = \sqrt[3]{x - 5}$

Since $f^{-1}(x)$ signifies the inverse function, we should use the algorithm of (1) switching *x* and *y* and then (2) solving for *y*. We are given that $y = x^3 + 5$ so step (1) gives $x = y^3 + 5$. Then solving for *y* gives $y^3 = x - 5$ so $y = \sqrt[3]{x-5}$. Thus, the inverse is given by $f^{-1}(x) = \sqrt[3]{x-5}$, option (d).

9. Write $\ln(x+1) - 3\ln(x+2)$ as a single logarithm.

(a) $\dfrac{\ln(x+1)}{\ln(x+2)^3}$ | (b) $\dfrac{x+1}{(x+2)^3}$

[(c)] $\ln\dfrac{x+1}{(x+2)^3}$ | (d) $\ln\frac{1}{8}$

Using the properties of logarithms,

$$\begin{aligned}\ln(x+1) - 3\ln(x+2) &= \ln(x+1) - \ln(x+2)^3 \\ &= \ln\frac{x+1}{(x+2)^3}. \quad \leftarrow \text{Answer}\end{aligned}$$

The answer is (c).

10. Find the slope of the line passing through the points $(1, -2)$ and $(-3, 4)$.

(a) 1 | (b) -1

(c) $\dfrac{3}{2}$ | [(d)] $-\dfrac{3}{2}$

The slope of the line is given by $m = \frac{\Delta y}{\Delta x} = \frac{4-(-2)}{-3-1} = \frac{6}{-4} = -\frac{3}{2}$. Choice (d) is the correct answer.