REFLECTIONS FROM TEACHING INQUIRY-ORIENTED DIFFERENTIAL EQUATIONS

2016 JOINT MATHEMATICS MEETINGS

Seattle, WA

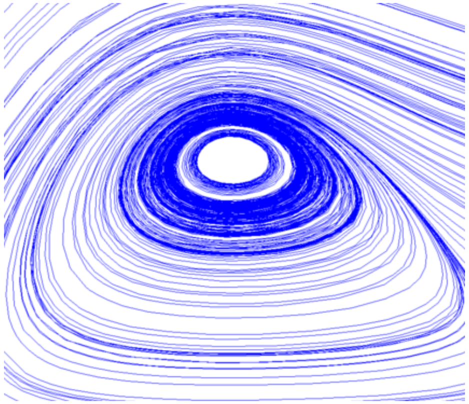
Jan 8, 2016

Keith Nabb

Moraine Valley Community College

Palos Hills, Illinois

nabb@morainevalley.edu



WHAT IS INQUIRY?

- Guiding students with questioning, prompting, and useful feedback
- Asking for reasoning and justification
- *Listening* to students: Using their ideas to teach
- Students take ownership of their learning and must make sense of new mathematics

WHAT IS IT NOT?

Sending students down aimless pathways

Hand-holding

Discovery learning

The teacher not “teaching”

INQUIRY IN A NUTSHELL…

1. Assign a recorder. Give the groups a task. Discuss the task to ensure everyone understands the purpose of the task as well as what they are required to produce.
2. Groups begin working. Teacher navigates from group to group, steering groups back on course if necessary.
3. Near the end of class, several groups present their findings. Students are encouraged to critique other groups' work and to ask questions.
4. Teacher makes remarks connecting the different presentations.
5. The next day, all classmates get a record of the day's events.

PREPARING STUDENTS FOR INQUIRY

MTH 201
Ice-breaker

"Map-Coloring Problem"

Question: What is the minimum number of colors sufficient to color a two-dimensional map? (Rules of the game are forthcoming)

1. Study a fictional map. Discuss boundaries and vertices.

2. Define what it means to "correctly" color a map (i.e., make up the rules).

3. Experiment a bit: For example, construct a map with 2 countries using 2 colors, 3 countries with 3 colors, 3 countries with fewer than 3 colors, etc.

4. Moving on: Can we construct a 4 country map with *fewer* than 4 colors? More challenging: Can we construct a 4 country map that *requires* 4 colors? Think about these questions & provide examples, if possible.

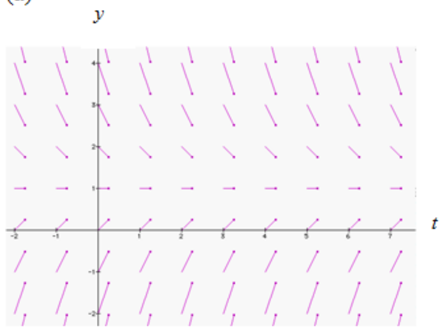
5. Natural extension: Move to 5 countries. It is possible to construct maps that only require 2, 3, or 4 colors to meet the requirements?

6. Challenge: Draw a 5 country map that *requires* 5 colors.

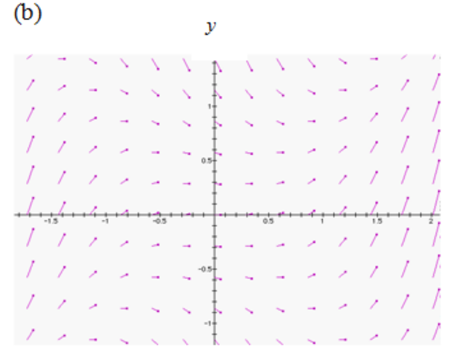
SAMPLE TASKS*

3. Below are three different tangent vector fields and six rate of change equations. Without using technology, identify which differential equation is the best match for each tangent vector field (Thus you will have three rate of change equations left over). Explain your reasoning.

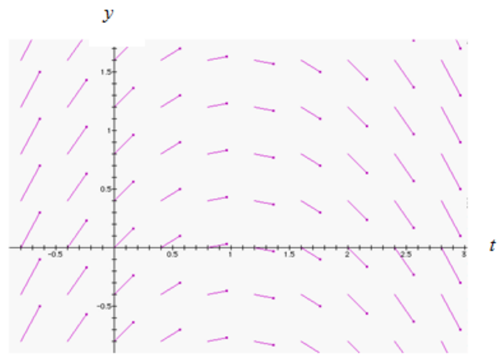
(a)



(b)



(c)



$$\frac{dy}{dt} = t - 1, \quad \frac{dy}{dt} = 1 - y^2, \quad \frac{dy}{dt} = y^2 - t^2, \quad \frac{dy}{dt} = 1 - y, \quad \frac{dy}{dt} = t^2 - y^2, \quad \frac{dy}{dt} = 1 - t$$

*Adopted from C. Rasmussen's IO-DE (Inquiry Oriented Differential Equations)

SAMPLE TASKS (continued)

Suppose two students are memorizing a list according to the rate of change equation $\frac{dL}{dt} = 0.5(1 - L)$, where L represents the fraction of the list that is memorized at any time t.

(a) If one of the students knows one-third of the list at time $t=0$ and the other student knows none of the list, which student is learning most rapidly at this instant?

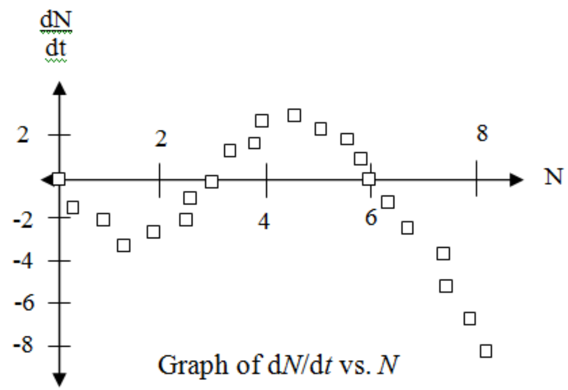
(b) According to the rate of change equation, will the student who starts out knowing none of the list ever catch up to the student who knows one-third of the list? Explain your reasoning.

(c) What does the rate of change equation predict for someone who starts off with the list completely memorized?

CLASSROOM EXAMPLES (task)

How Many Bugs?

A group of biologists is studying a particular bug population in a rainforest. They gathered data about these bugs for different population values, N, at different times, t. The scientists reasoned that the rate of change depended only on the population and not on time. They approximated the derivatives $\frac{dN}{dt}$ (as was done with the cooling coffee from before) and plotted $\frac{dN}{dt}$ versus N, as seen below:

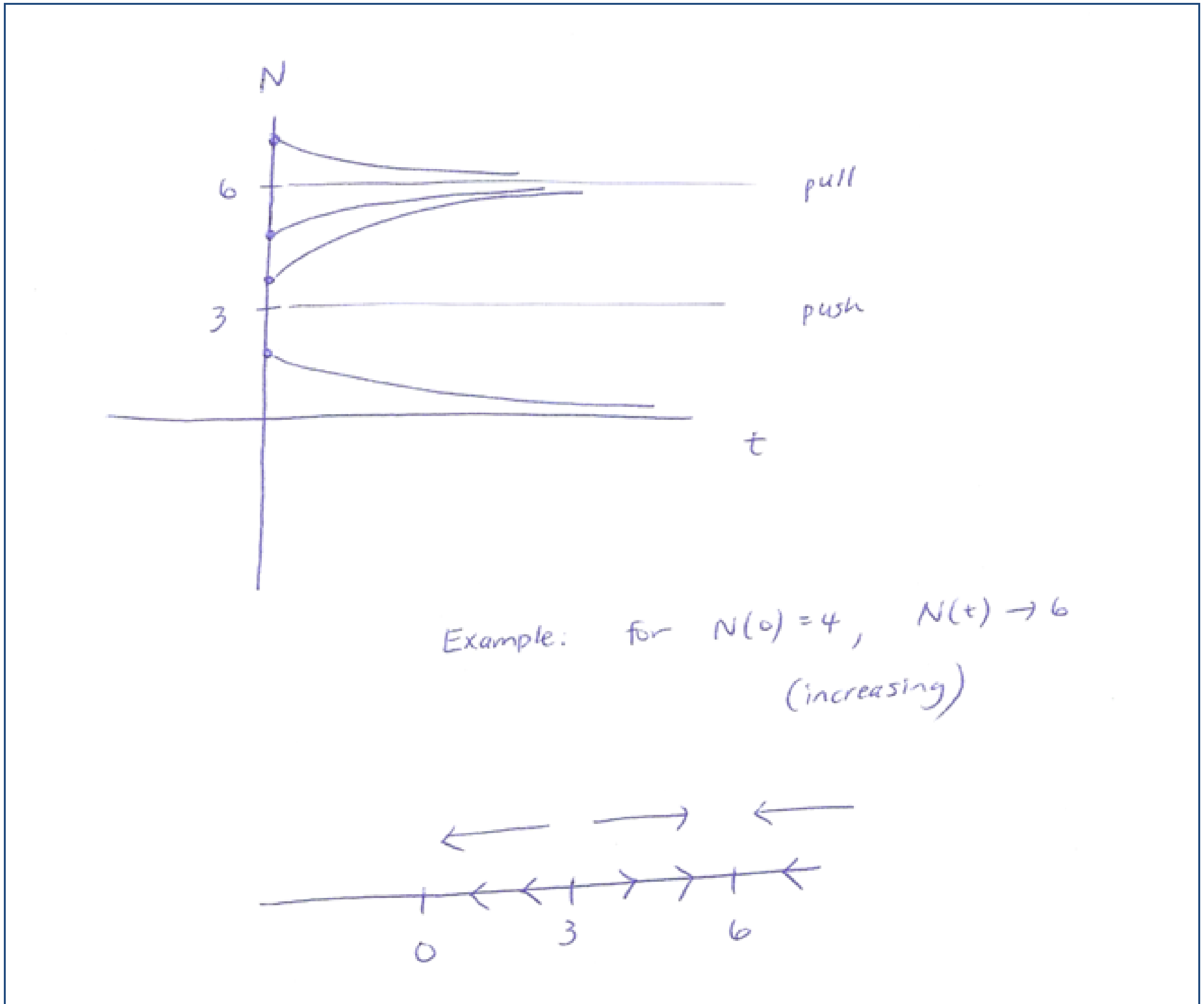


Graph of d*N*/d*t* vs. *N*

For the following initial population of bugs, use the above graph to predict what the ultimate fate of the population will be. Describe (in words) the long-term behavior of each solution corresponding to the given initial condition. In addition, illustrate your conclusions with a suitable graph.

a) $N(0) = 2$
b) $N(0) = 3$
c) $N(0) = 4$
d) $N(0) = 4.5$
e) $N(0) = 6$
f) $N(0) = 8$

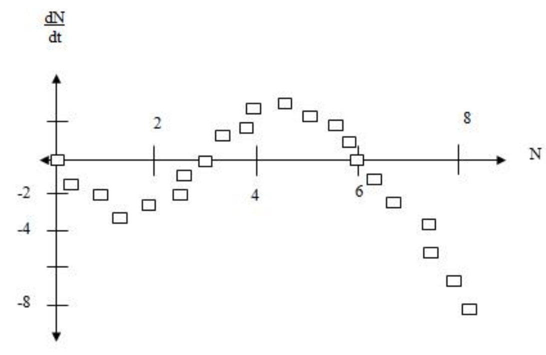
CLASSROOM EXAMPLES
(student work)



CLASSROOM EXAMPLES (student work--recorder)

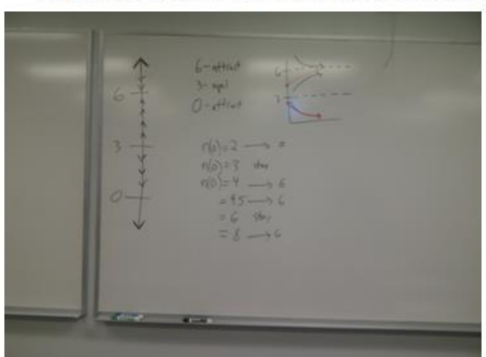
Ashley's Notes
6/30/13

How Many Bugs?
This exercise gave us another opportunity to work with more autonomous DEs. It allowed us to have a better understanding of stationary points and whether they were considered a repeller or an attractor.
This is the first graph that was given:



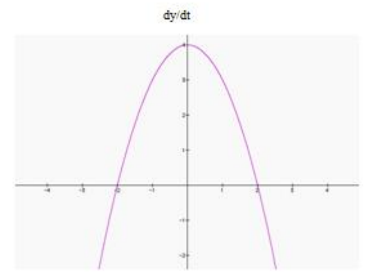
Graph of d*N*/d*t* vs. *N*

Brower showed on the board that the stationary points of this DE can be found where the graph crosses the N axis. In this case, the stationary points are 0, 3, and 6. Making a flow chart, it is easy to see that the points 0 and 6 are attractors and the point 3 is a repeller. The only place where there is growth in this situation, the "sweet spot," is between the points 3 and 6. Using the graph itself, this can be found wherever dN/dt is positive. Using the flow chart we can assume the long term behaviors of these initial conditions are as follows:



N(0)= 2 The population will approach 0
N(0)= 3 The population will remain at 3
N(0)= 4 The population will approach 6
N(0)= 4.5 The population will approach 6
N(0)= 6 The population will remain at 6
N(0)= 8 The population will approach 6

The second situation, involves this graph:



Matt showed that the stationary points in this graph were at y=-2 and 2. Just like the previous problem, these points can be found where the graph crosses the y-axis. Using a flow chart, it can be seen that the point -2 is a repeller, and the point 2 is an attractor. This would mean that the "sweet spot," the spot in which there is population growth, is between -2 and 2.

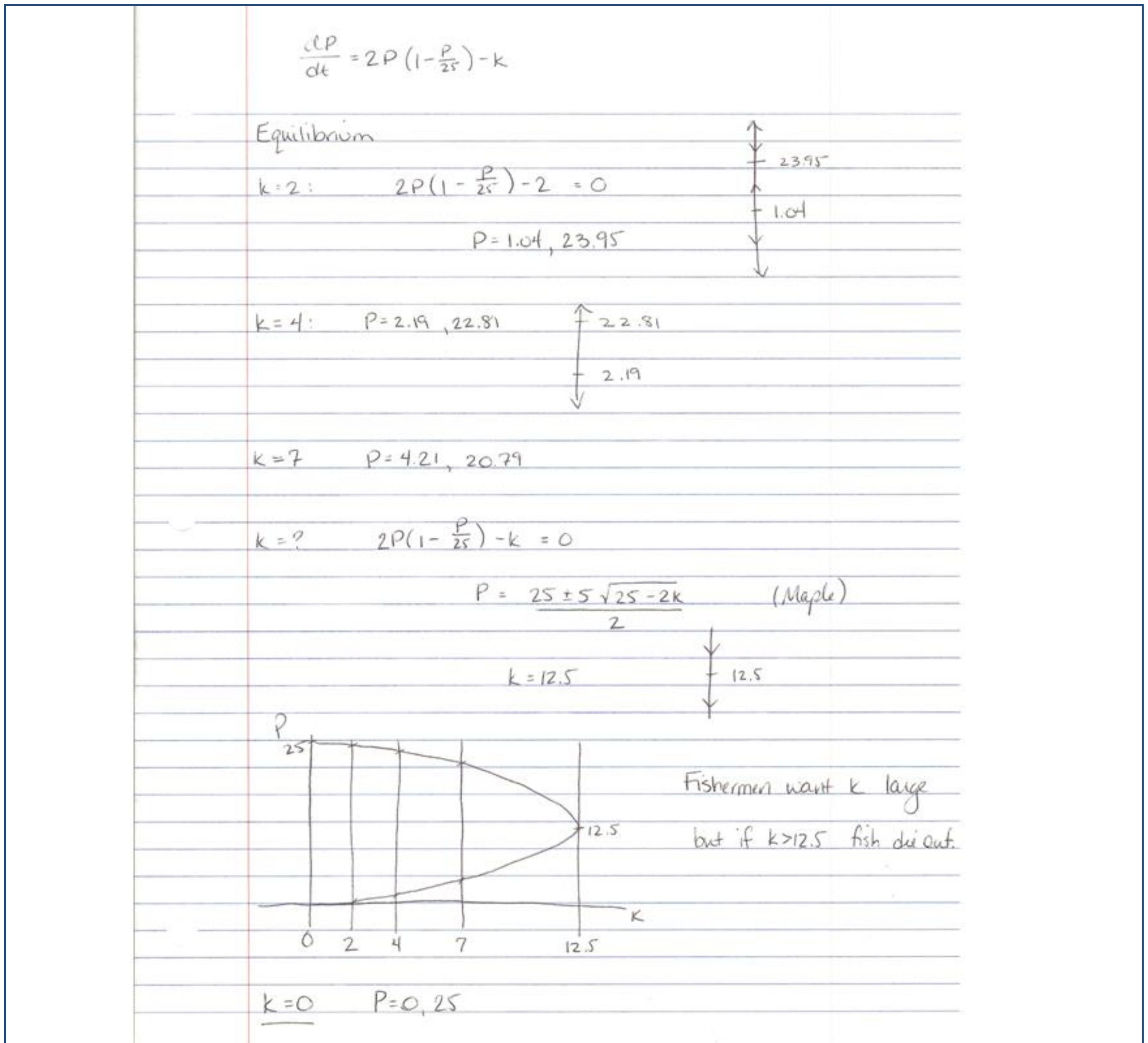
CLASSROOM EXAMPLES (task)

Fish Harvesting

A scientist at a fish hatchery has previously demonstrated that the rate of change equation $\frac{dP}{dt} = 2P(1 - \frac{P}{25})$ is a reasonable model for predicting the number of fish that the hatchery can expect to find in their pond.

Recently, the hatchery was bought out by fish.com and the new owners are planning to allow the public to catch fish at the hatchery (for a fee of course). The new owners need to decide how many fish per year they should allow to be harvested. Prepare a report for the new owners that illustrate the implications that various choices for harvesting will have on future fish populations.

CLASSROOM EXAMPLES (student work)



$$\frac{dP}{dt} = 2P\left(1-\frac{P}{25}\right) - k$$

Equilibrium

$k=2$: $2P\left(1-\frac{P}{25}\right) - 2 = 0$

$P = 1.04, 23.95$

$k=4$: $P = 2.19, 22.81$

$k=7$ $P = 4.21, 20.79$

$k=?$ $2P\left(1-\frac{P}{25}\right) - k = 0$

$$P = \frac{25 \pm 5\sqrt{25-2k}}{2}$$ (Maple)

$k = 12.5$

Fishermen want k large but if $k > 12.5$ fish die out.

$k=0$ $P = 0, 25$

BARRIERS TO INQUIRY

1. STUDENTS
2. TIME
3. ASSESSMENT OF KNOWLEDGE
4. TEACHERS

ASSESSMENT IS MESSY

1. Is the mathematics produced correct?
2. Are the assumptions valid?
3. Can the students defend their solution(s) and communicate important mathematical ideas to each other and to the class? Are the groups able to answer questions from classmates?
4. Since not all groups present on any given day, different groups may have grades for a particular assignment while others may not!

STUDENT FEEDBACK

- “I like this class because it makes me think more. It’s different from other math classes that I took before.”
- “This is sweet as hell. Instead of telling us how to think you let us figure out how to approach the problems.”
- “I’ve learned never to give up. In the beginning we are usually clueless but by the end of the period, we are good to go. I’ve noticed this has carried over into my homework. If I see something I think I can’t do, it doesn’t scare me. I attack it because I know I can do it.”
- “Besides the math, I’m learning that math is not what I thought it was. There are so many logical ways to approach problems and seeing other people’s work has been a real eye-opener for me.”
- “I actually remember this stuff later on and I’m not bored.”

ADDITIONAL BENEFITS

1. Students bolster their presentation and communication skills
2. Students learn to be free thinkers
3. Students get a chance to critique the work of others
4. Students catch a glimpse into the nature of mathematics
5. Students gain valuable experience in a teamwork setting
6. Certain students may thrive in this setting

Questions?

nabb@morainevalley.edu