June 13, 2017

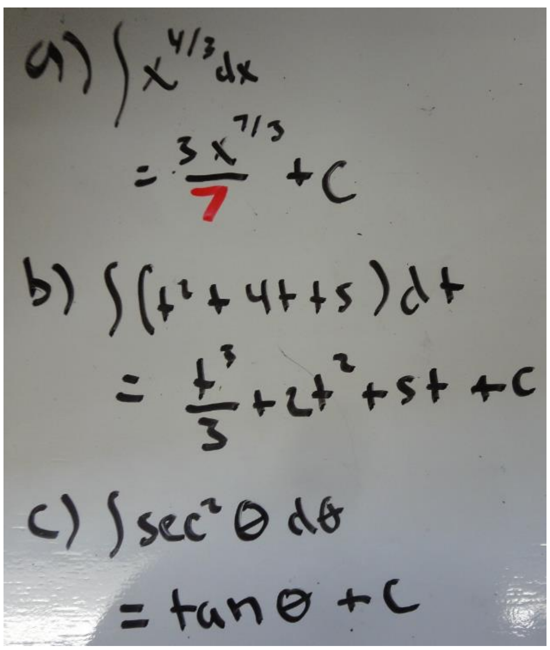
4.4: Indefinite Integrals & the Net Change Theorem

PROBLEM (LEVEL I)

Evaluate.

(a) $\int x^{4/3}dx$ (b) $\int \left(t^2+4t+5\right)dt$

(c) $\int \sec^2\theta\, d\theta$



PROBLEM (LEVEL II)

Evaluate.

(a) $\int (3x+5)^2\, dx$ (b) $\int \frac{x^3+4x}{x}dx$

(c) $\int \sin(3x)dx$

A) $\int (3x+5)^2\,dx =$

$(3x+5)(3x+5) = \underline{9x^2 + 30x + 25}$

$F(x) = 3x^3 + 15x^2 + 25x + C$

B.) $\int \frac{x^3+4x}{x}\,dy$

$= \int \left(\frac{\cancel{x^3}^{\,2}}{\cancel{x}} + \frac{4\cancel{x}}{\cancel{x}} \right) dx$

$= \int (x^2 + 4)\,dx$

$= \frac{1}{3}x^3 + 4x + C$

C. $\int \sin(3x)\,dx = \boxed{-\frac{1}{3}\cos 3x + c}$

$-\cos\left(\frac{3}{2}x^3\right)$ ⇦ **incorrect**

$\frac{d}{dx}\left(-\frac{1}{3}\cos(3x)\right) = \sin(3x)$

$\cos 3x$

$-\cos 3x$

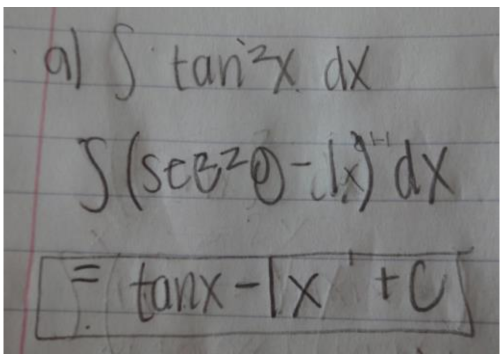
$-\frac{1}{3}\cos 3x$

PROBLEM (LEVEL III)

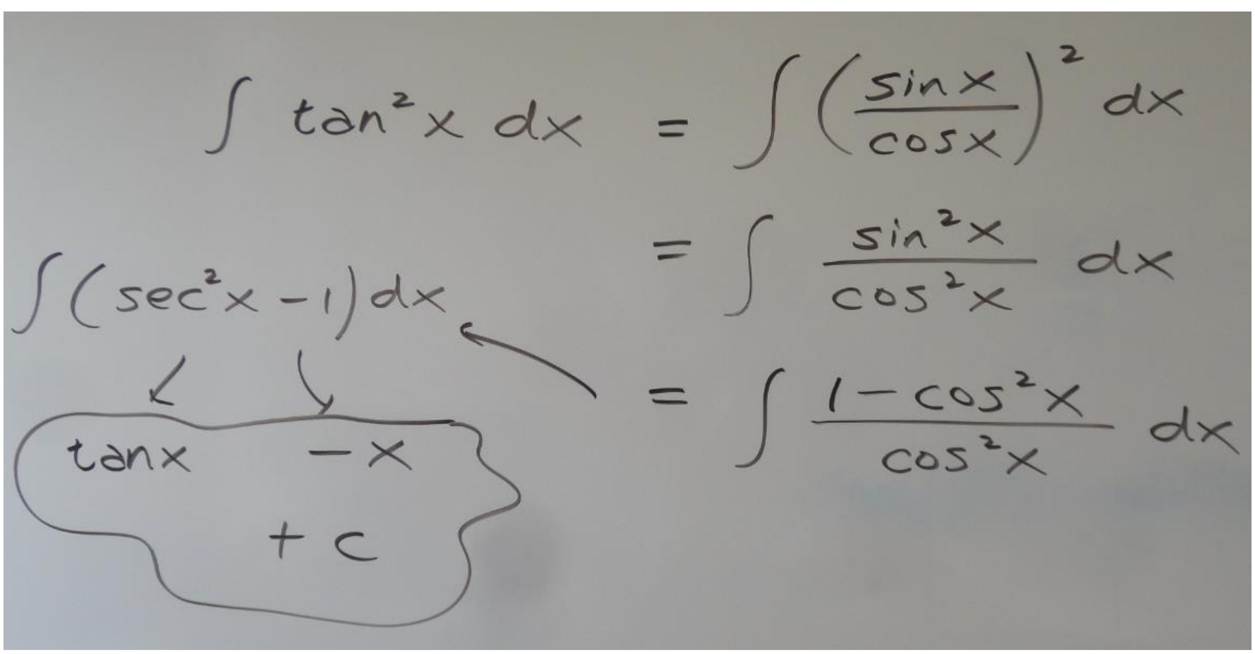
Evaluate.

(a) $\int \tan^2 x\, dx$ (b) $\int \frac{1}{1-\cos x}\, dx$

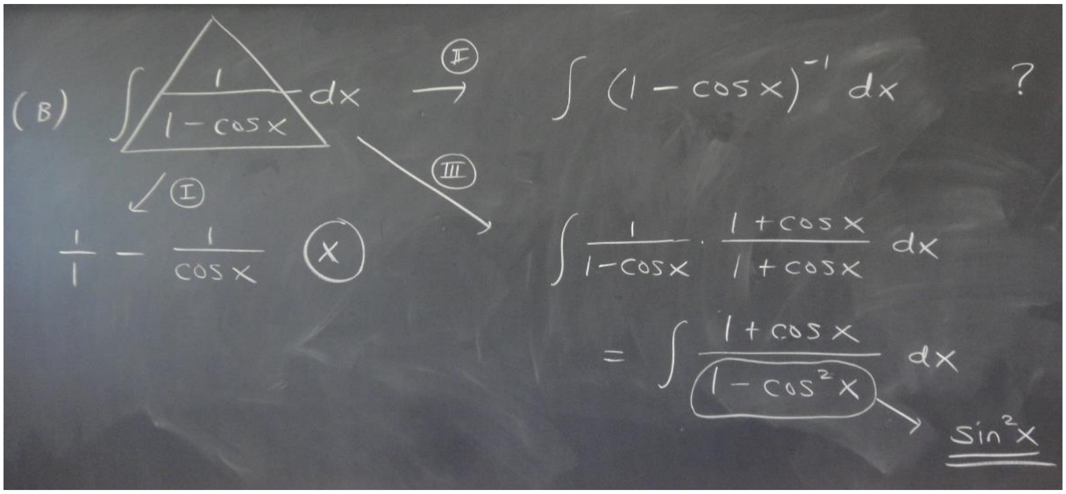
Problem A:

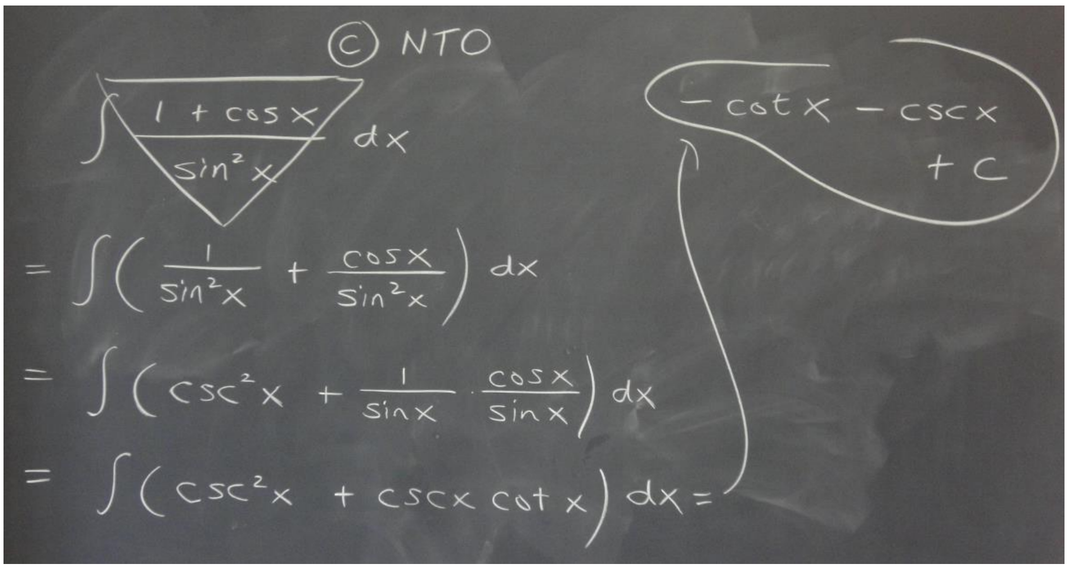


The above notebook work is great as long as we recall the trig identity $\tan^2 x = \sec^2 x - 1$. What if you don't remember it? See below (using the definition of tangent along with the easier-to-recall identity $\sin^2 x + \cos^2 x = 1$ might be a better route). Either way leads to success.



Problem B:





FUNDAMENTALS

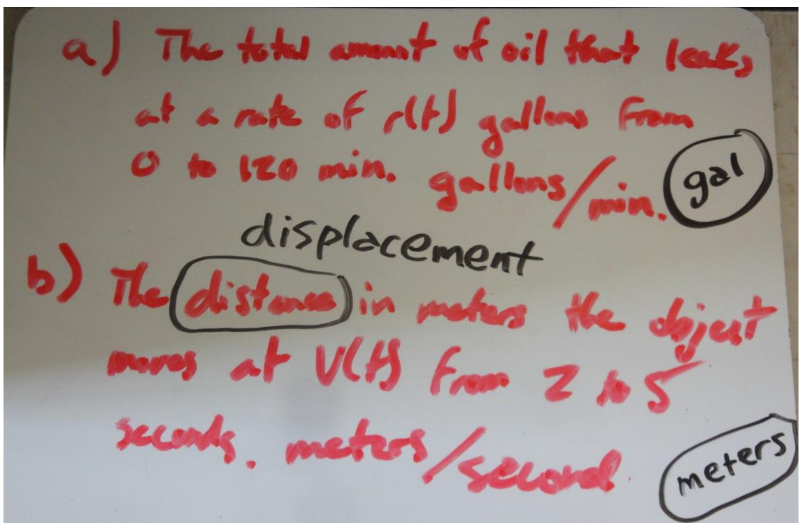
$$\int_a^b Q'(x)\,dx = Q(b) - Q(a)$$

[Net Change in Q]

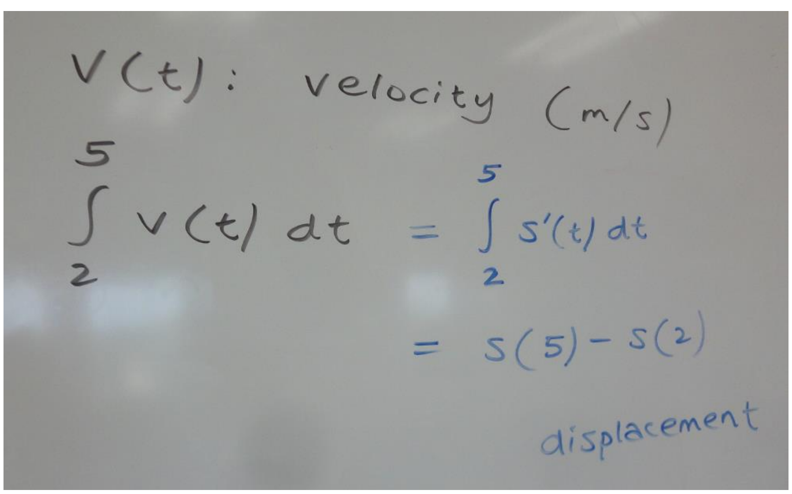
"THINKING" PROBLEMS

(a) Oil leaks from a tank at the rate of $r(t)$ gallons per minute at time t. What is the meaning of $\int_0^{120} r(t)\,dt$? What are its units?

(b) If an object moves along a straight path with velocity $v(t)$ meters/sec, what is the meaning of $\int_2^{5} v(t)\,dt$? What are its units?



Supplemental work for part (b):

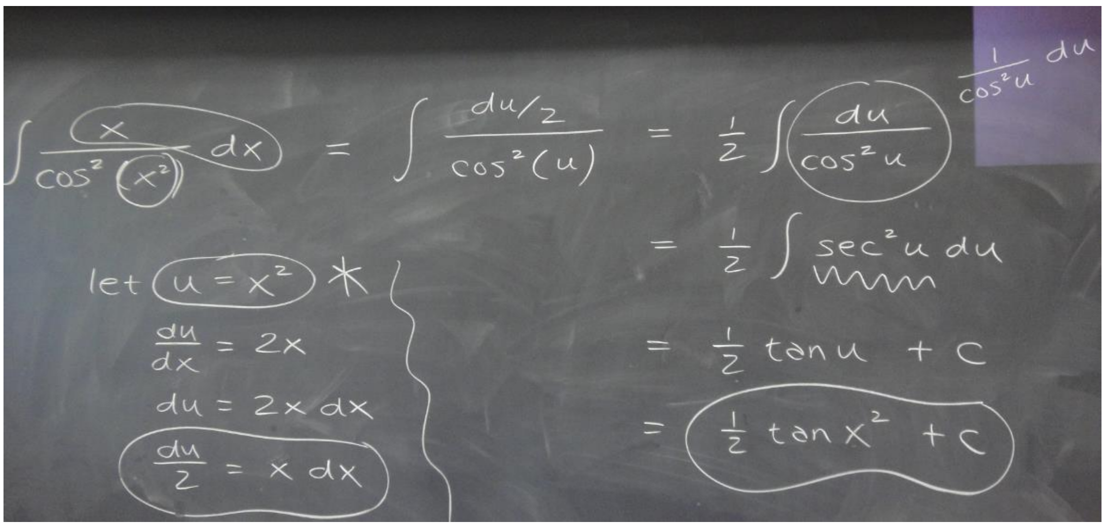


4.5: Substitution

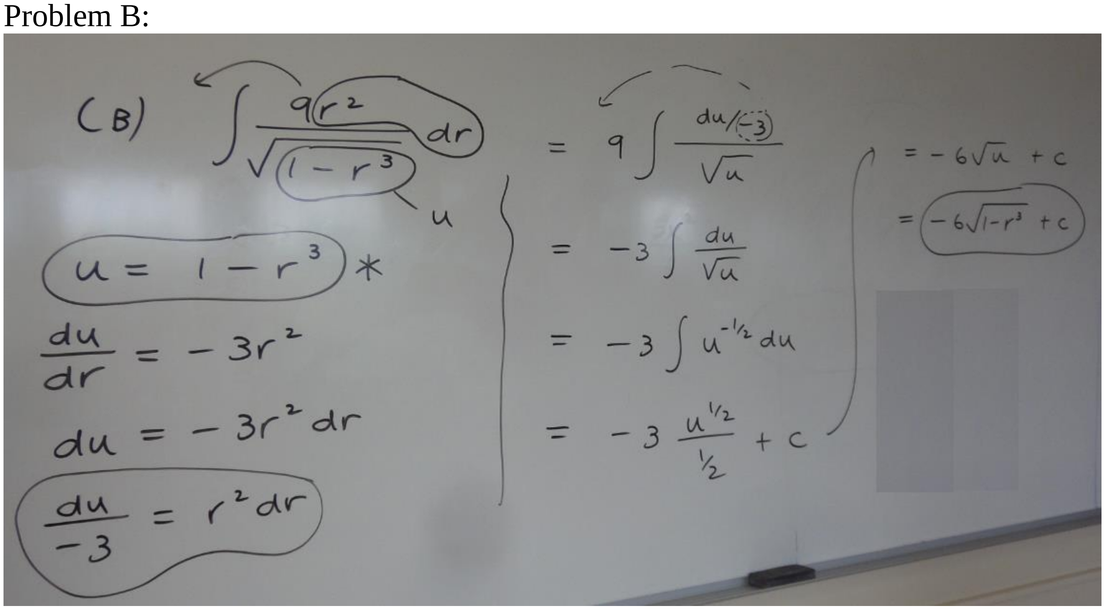
PROBLEMS

(a) $\displaystyle\int \frac{x}{\cos^2\left(x^2\right)}\,dx$ (b) $\displaystyle\int \frac{9r^2}{\sqrt{1-r^3}}\,dr$

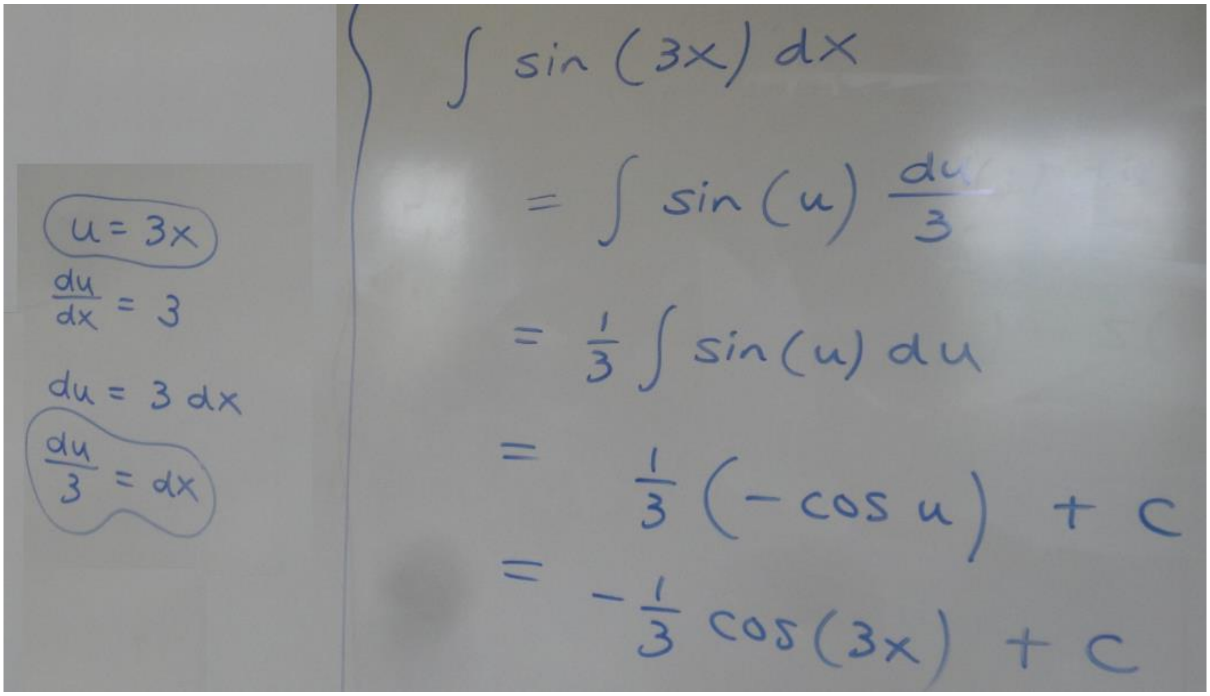
Problem A:



Problem B:



Revisiting the problem $\int \sin 3x\, dx$, now with substitution:



PROBLEMS

(a) $\int \sin^3 x \cos x\, dx$

(b) $\int \sec(2z)\tan(2z)\,dz$

(c) $\int \sec^2\theta \tan^4\theta\, d\theta$

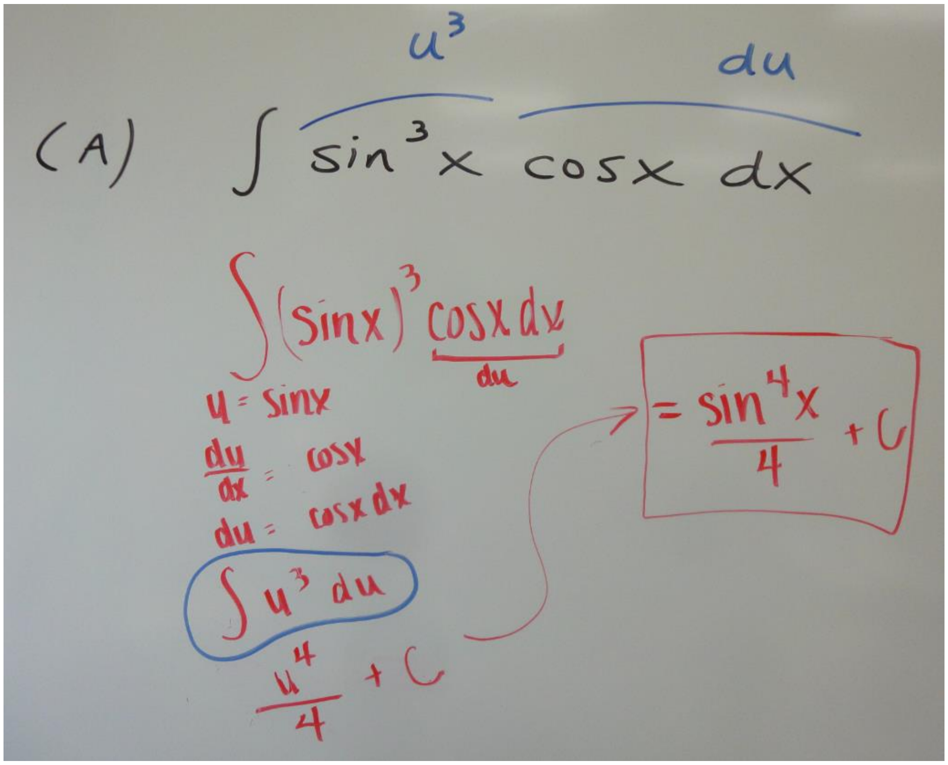
(d) $\int v^5\sqrt{1+v^2}\,dv$

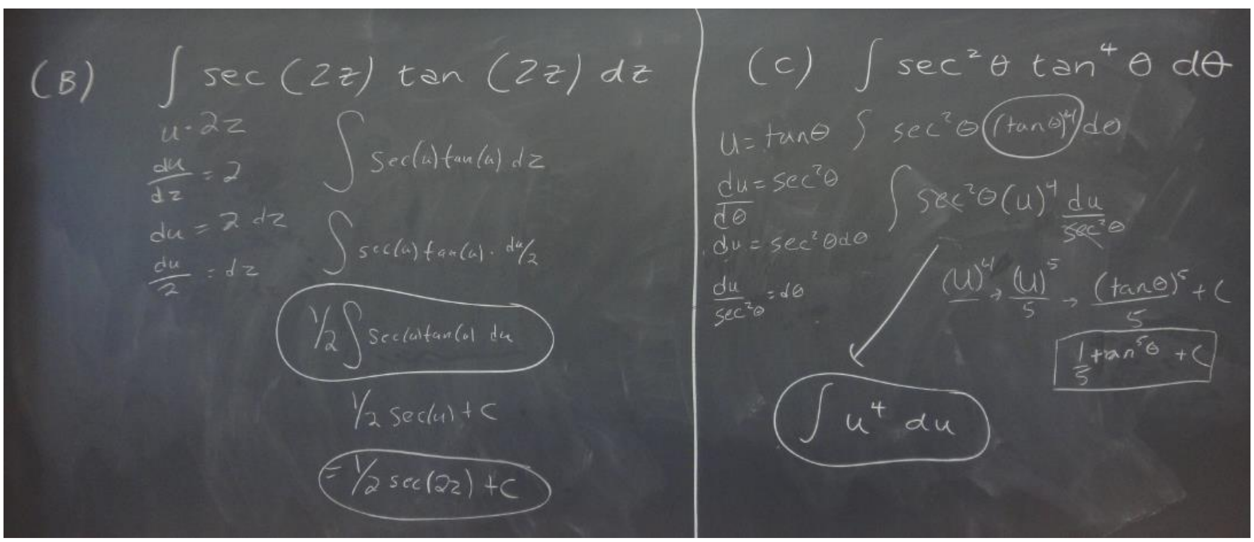
(e) $\int \frac{\cos\left(\frac{1}{\theta}\right)}{\theta^2}\,d\theta$

(f) $\int x\sqrt{9-x^2}\,dx$

(g) $\int (3x+4)^{10}\,dx$

For each problem below, the step showing the transformation from the original variable (x, z, θ, v) to the u variable is circled/boxed. In each case, the problem in variable u is **easier** than the problem originally posed (and this is basically the point of u-substitution).





(d) coming later…

(E) $\int \frac{\cos(1/\theta)}{\theta^2}\, d\theta$

$u = 1/\theta$ or θ^{-1}

$\frac{du}{d\theta} = -1\theta^{-2}$

$du = -1\theta^{-2}\, d\theta$

$-du = \theta^{-2}\, d\theta$ or $\frac{1}{\theta^2}\, d\theta$

$\int \frac{\cos(u)}{\theta^2}\, d\theta$

$\int \cos(u) \cdot -du$

$-\sin(u) + C$

$= -\sin(1/\theta) + C$

*

(F) $\int x\sqrt{9-x^2}\, dx$

$u = 9 - x^2$

$\frac{du}{dx} = -2x$

$du = -2x\, dx$

$\frac{du}{-2} = x\, dx$

$\int \sqrt{u} \left(\frac{du}{-2}\right)$

$-\frac{1}{2}\int \sqrt{u}\, du$

$\frac{-1}{2}\int u^{1/2}\, du$

$\frac{-1}{2}\,\frac{u^{3/2}}{3/2}$

$-\frac{1}{3}\sqrt{u}$

$= \frac{-1}{3}\sqrt{9-x^2} + C$

(G) $\int (3x+4)^{10}\, dx$

$u = 3x+4$

taking derivative

$\frac{du}{dx} = 3$

$du = 3 \cdot dx$

$\frac{du}{3} = dx$

$\int (u)^{10}\, \frac{du}{3}$

moved $1/3$

took integral of function (leaving u alone)

$\frac{1}{3}\,\frac{(u)^{11}}{11} + C$

$\frac{1}{3}\,\frac{(3x+4)^{11}}{11} + C$

$\frac{(3x+4)^{11}}{33} + C$