

June 2, 2017

3.2: Mean Value Theorem

TWO IMPORTANT THEOREMS

Let f be continuous and smooth on the interval $[a,b]$. Then...

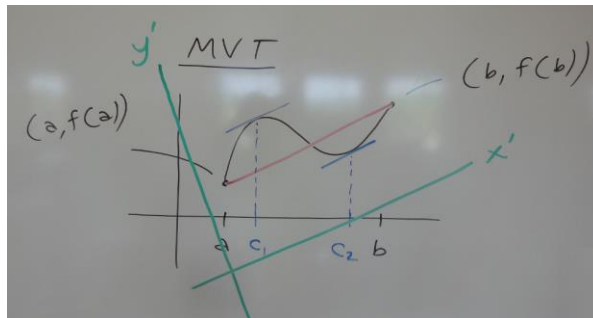
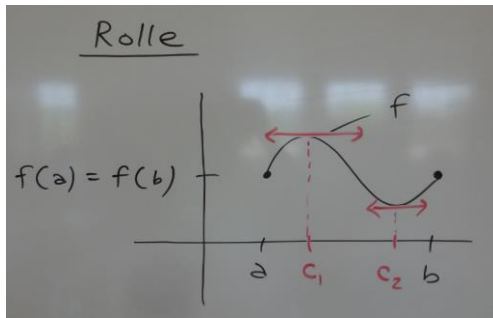
Rolle's Theorem

... if $f(a) = f(b)$, there exists a number c in (a,b) such that $f'(c) = 0$.

Mean Value Theorem

... there exists a number c in (a,b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Visual thinking for Rolle's Theorem & the Mean Value Theorem. Notice that Rolle's Theorem is a special case of the more general MVT.



IROC

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

AROOC

slope of TL at $x=c$

slope of secant line (through endpoints)

← MVT

WARM UP

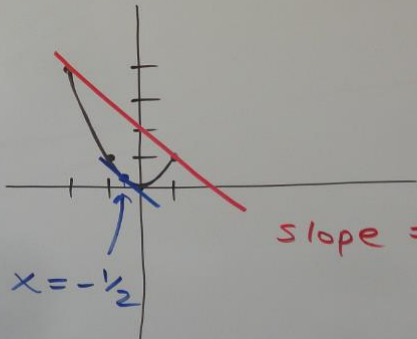
Apply the Mean Value Theorem
to $f(x) = x^2$ on the interval
 $[-2, 1]$. If possible, find the c
value and explain what it means.

Use MVT on
 $[a, b]$
 $f(x) = x^2$ on $[-2, 1]$.
Find c ; explain.
 $f'(x) = 2x$
 $f'(c) = 2c$

* $2c$

$$f'(c) = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{1^2 - (-2)^2}{1 + 2} = -1$$

$2c = -1$
 $c = -\frac{1}{2}$



The graph shows the parabola $f(x) = x^2$ on the interval $[-2, 1]$. A red secant line connects the points $(-2, 4)$ and $(1, 1)$. A blue tangent line is drawn at the point $(-\frac{1}{2}, \frac{1}{4})$. The slope of the secant line is -1 , and the slope of the tangent line is also -1 . The x-axis is labeled with $x = -\frac{1}{2}$ at the point of tangency.

slope = -1

PROBLEM

Two stationary patrol cars are 5 miles apart on a highway. As a truck passes the first patrol car, its speed is clocked at 55 mph. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50 mph. Prove that the truck must have exceeded the speed limit (>55 mph) at some time during the trip.

The diagram shows a horizontal line representing a highway. Two points on the line are marked with boxes containing an 'X', representing patrol cars. A double-headed arrow between these points is labeled '5 mi'. Below the line, a dashed line represents the truck's path, with a small rectangle and an arrow pointing to the right. Above the line, the text 't=0' and '55 mph' is written on the left, and 't = 4 min' and '50 mph' is written on the right. To the right of the diagram, the following calculations are shown:

$$D = RT$$
$$R = \frac{D}{T} = \frac{5 \text{ mi}}{4 \text{ min}}$$
$$= \frac{5 \text{ mi}}{\frac{4}{60} \text{ hr}}$$
$$= \frac{5 \text{ mi}}{\frac{1}{15} \text{ hr}}$$
$$= 75 \text{ mph}$$

The above work tells us that the truck driver's *average velocity* between the officers was 75 mph. Using the MVT idea (the rate of change interpretation), **there must have been an instant somewhere between the officers where the driver was traveling exactly 75 mph** (instantaneous rate of change). We don't know where this occurred but the MVT guarantees it occurred somewhere during the journey. There is clear evidence to support a speeding ticket.