Reflection 3.3
June 5

This lesson was broken apart into three sections. The first section was the First Derivative Test, second was Concavity and Points of Inflection, and last was the Second Derivative Test. All of these sections build on each other and information used in the Second Derivative Test will come from the previous sections.

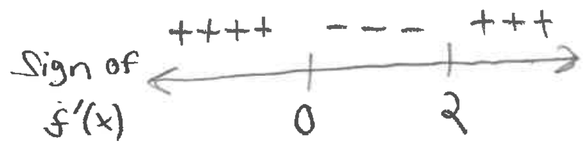
The first section was the First Derivative Test. The derivative of the function needed to be found and from the derivative the critical numbers can be found by setting the derivative to zero. Then a number line is helpful using the critical numbers to determine if the tangent line has a positive or negative slope. Any number in between and outside of the critical numbers are used and evaluated in the original function to find a positive or negative slope for that area.

$$f(x) = x^3 - 3x^2 + 3$$
$$f'(x) = 3x^2 - 6x$$
$$= 3x(x-2)$$

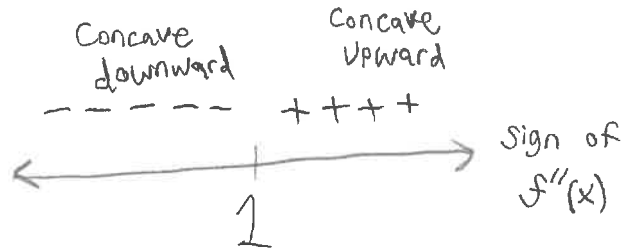
Critical Numbers

0, 2

Number line using Critical Numbers



In the Concavity section we needed to find the second derivative of a function and set it to zero. By doing this we are able to find possible points of inflection. Again, we put these points on a number and determine a number in between/outside of the possible points of inflection and decide if it returns a positive or a negative number. If it returns a negative number then it has a concave downward, if it returns a positive number it has a concave upward. The points of inflection are determined where the graph switches from one concave to the other and if the limit exist at that point.



$$f(x) = x^3 - 3x^2 + 3$$
$$f'(x) = 3x^2 - 6x$$
$$f''(x) = 6x - 6$$

Possible Point of Inflection

1

$$f''(x) = 6x - 6$$

Concave Upward: $(1, \infty)$

Concave downward: $(-\infty, 1)$

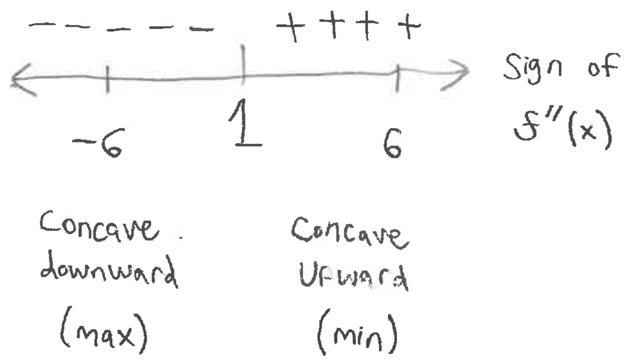
For the final section, Second Derivative Test, all of the information that was found in the previous two parts is needed to find the relative minimum and relative maximum of the function. To do this the critical numbers found in the derivative are needed and must be evaluated in the second derivative. After we solve the second derivative by using the critical numbers, the number line used in to define concavity is used to find if it is a concave upward or downward at the point just found. If it has concave downward it is a relative maximum value and if it has a concave upward then it is a relative minimum value. Lasty, to find the y values the critical numbers need to be solve the original function using the critical numbers..

$f''(x) = 6x - 6$

Critical Numbers found in $f'(x)$: 0, 2

$$f''(0) = 6(0) - 6$$
$$= -6$$

$$f''(2) = 6(2) - 6$$
$$= 6$$



Plug in critical Numbers in Original function to find the y values

$f(x) = x^3 - 3x^2 + 3$

$$f(0) = (0)^3 - 3(0)^2 + 3$$
$$= 3$$

$\longrightarrow$ Maximum: (0, 3)

$$f(2) = (2)^3 - 3(2)^2 + 3$$
$$= -1$$

$\longrightarrow$ Minimum: (2, -1)