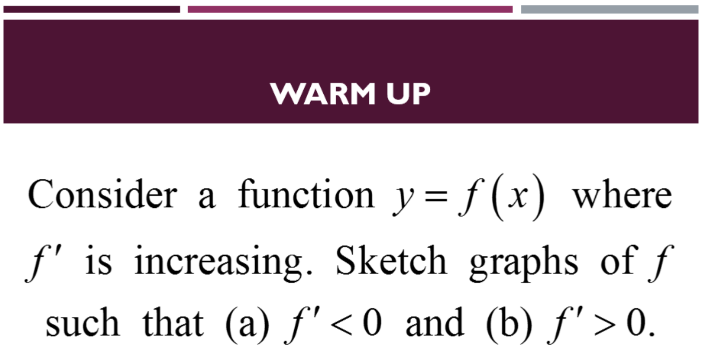
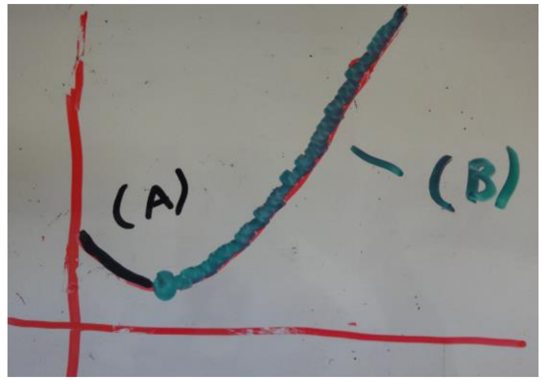
June 5, 2017

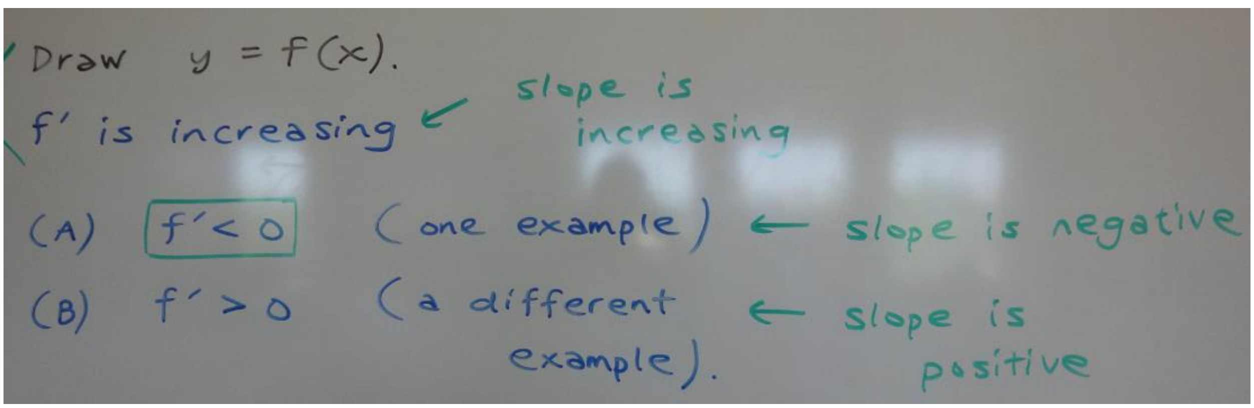
3.3: How Derivatives Affect the Shape of the Graph

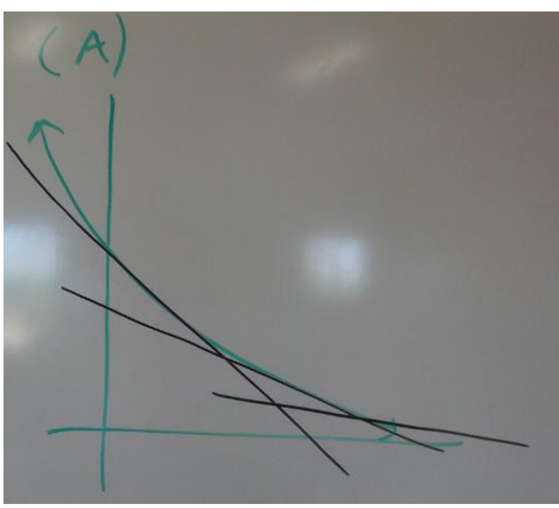


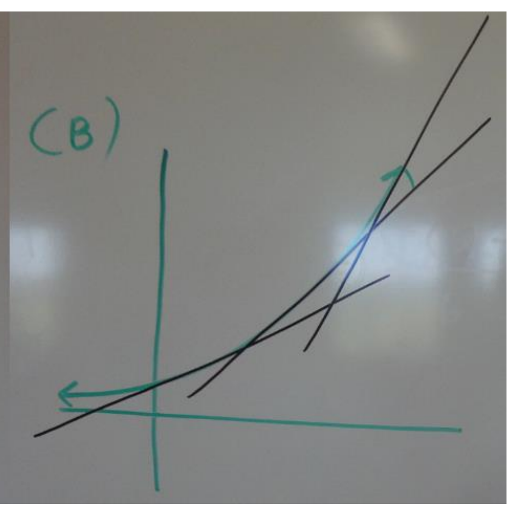


The graph (above right) has both characteristics (see the labeling directly on the graph).

A great way to tackle a problem like this is to (a) translate/decode the meaning in the statements (see board below) and then (b) use that information to make a sketch.







PROBLEM

For each of the problems below, do the following:

(a) Find the critical number(s).

(b) State the intervals of increase/decrease.

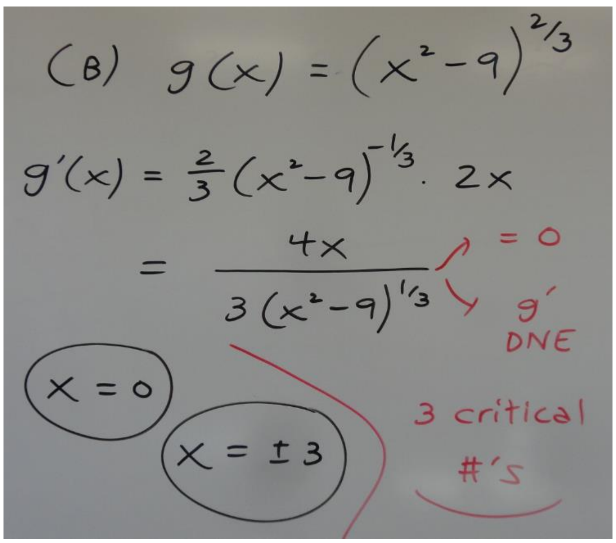
(c) Use the FDT to classify the extreme values.

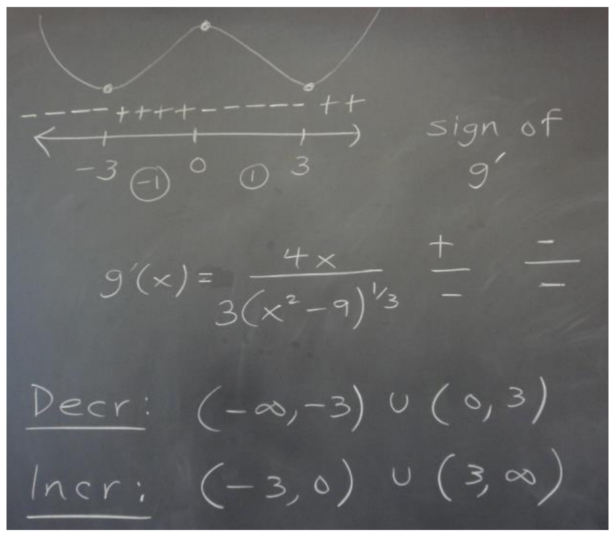
(d) Use your calculator to confirm your findings.

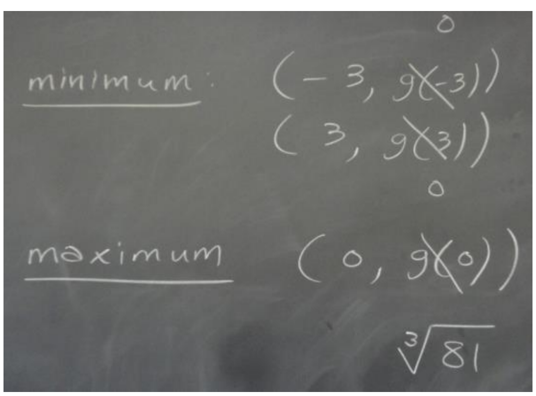
(a) $f(x) = x^2 - 4x$ (b) $g(x) = (x^2 - 9)^{2/3}$

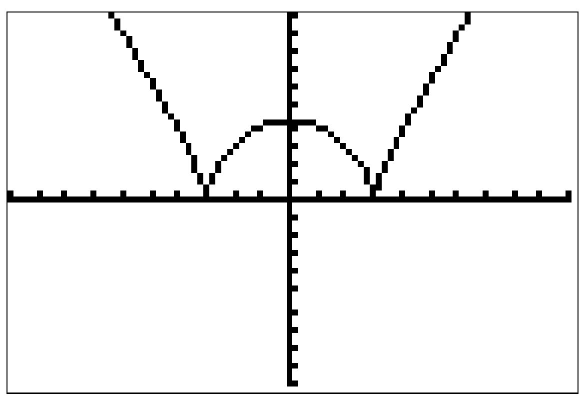
(c) $y = x\sqrt{6 - x}$ (d) $h(x) = x + 2\sin x, \quad 0 \le x \le 2\pi$

We did problem (b) together:



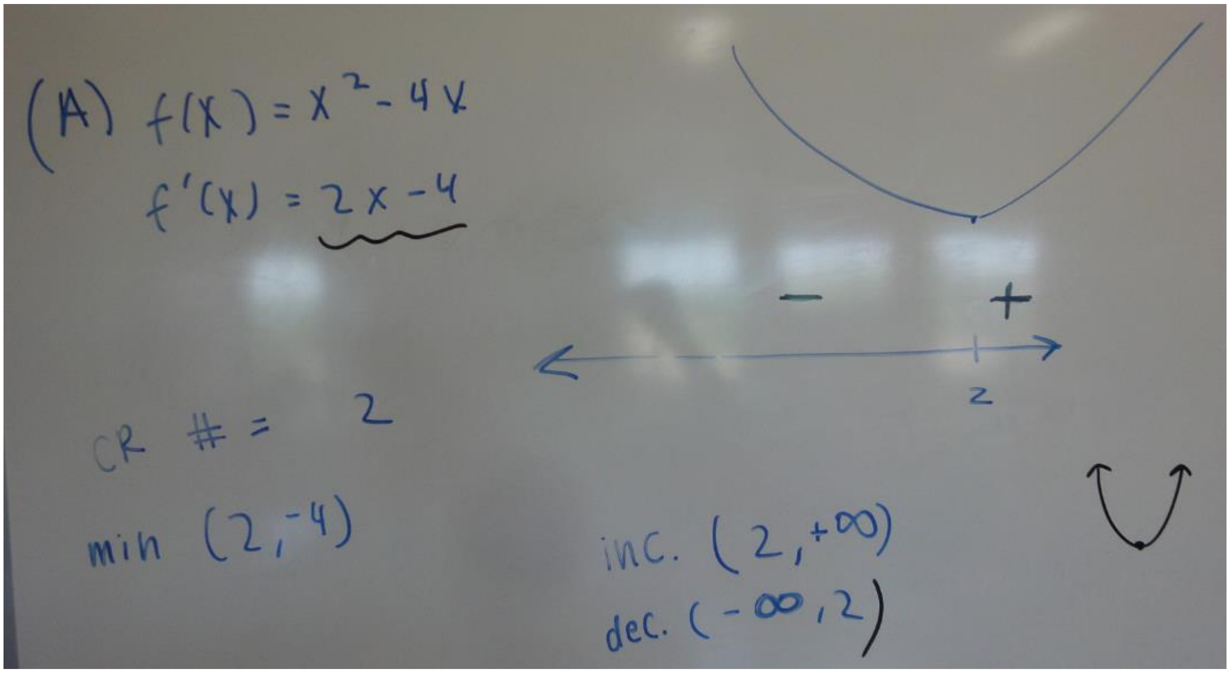




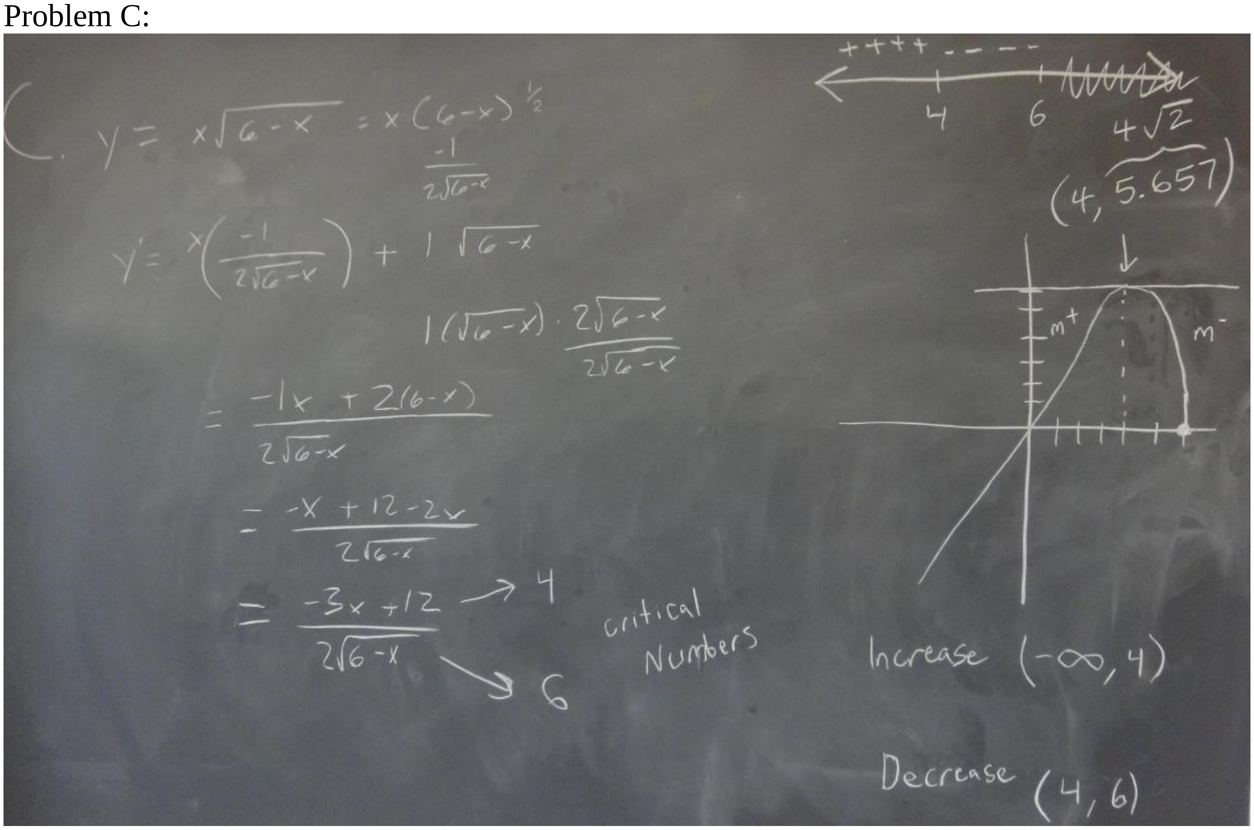


From this, we see minima at $(\pm 3, 0)$ and a maximum at $(0, \sqrt[3]{81})$. Notice the graph also shows the cusps at $(\pm 3, 0)$—where $g(x)$ is not differentiable (not smooth).

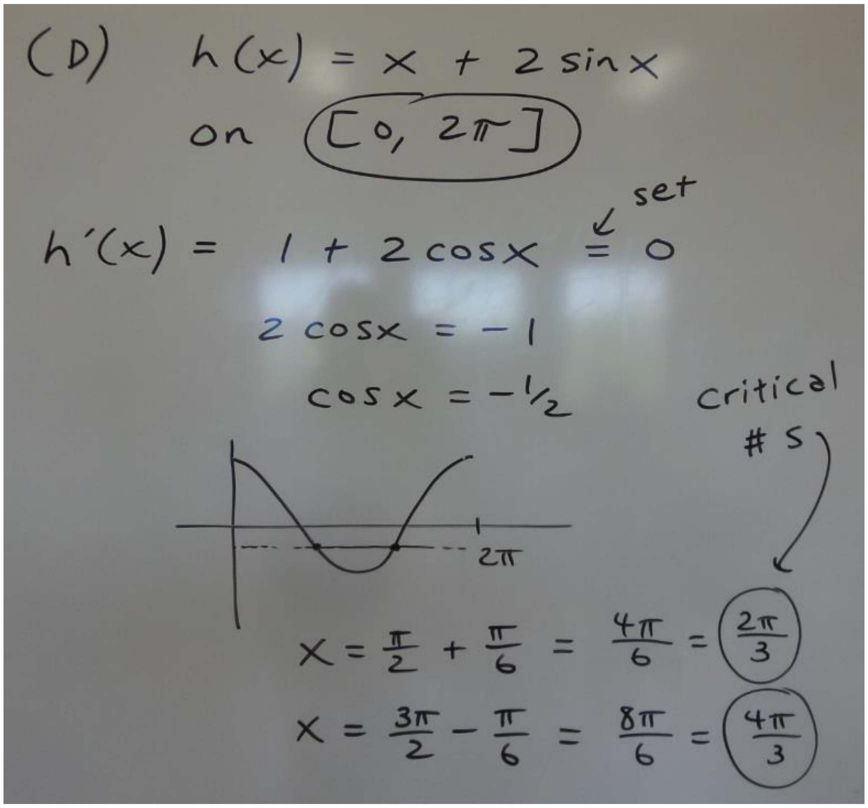
Problem A:

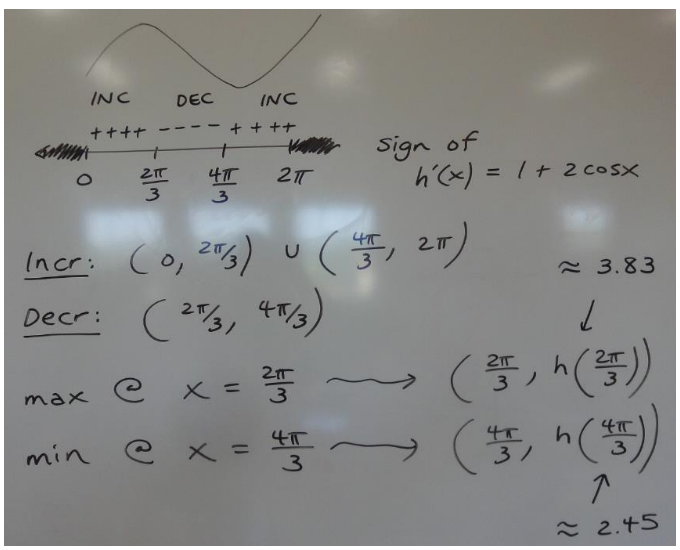


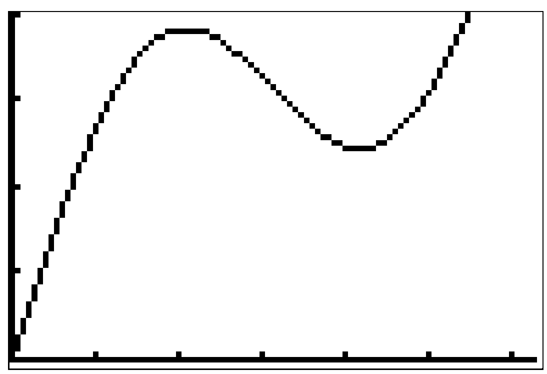
Problem C:



Problem D:







Notice the above mathematical analysis identifies the hill and valley in the graph. The **absolute** min/max appear to be different from these values (absolute min at $(0,0)$ [lower left of the graph] and the absolute max at $(2\pi,2\pi)$ [upper right of the graph]).

PROBLEM

For each of the problems below, do the following:

(a) Discuss the concavity of the graph.

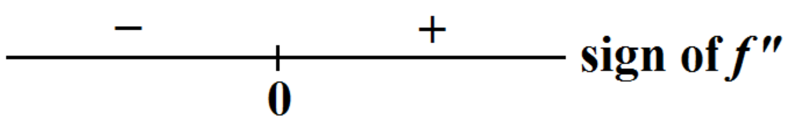
(b) Find any points of inflection.

(c) Confirm the above on a calculator.

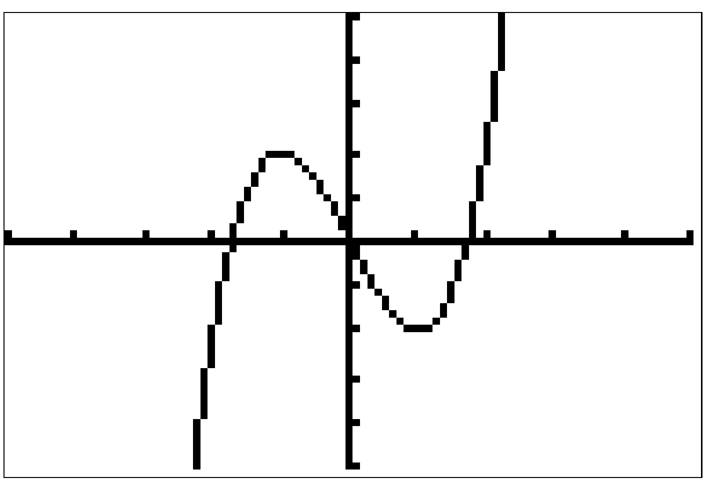
$$\text{(a) } f(x) = x^3 - 3x$$

$$\text{(b) } y = x^4 - 4x^3$$

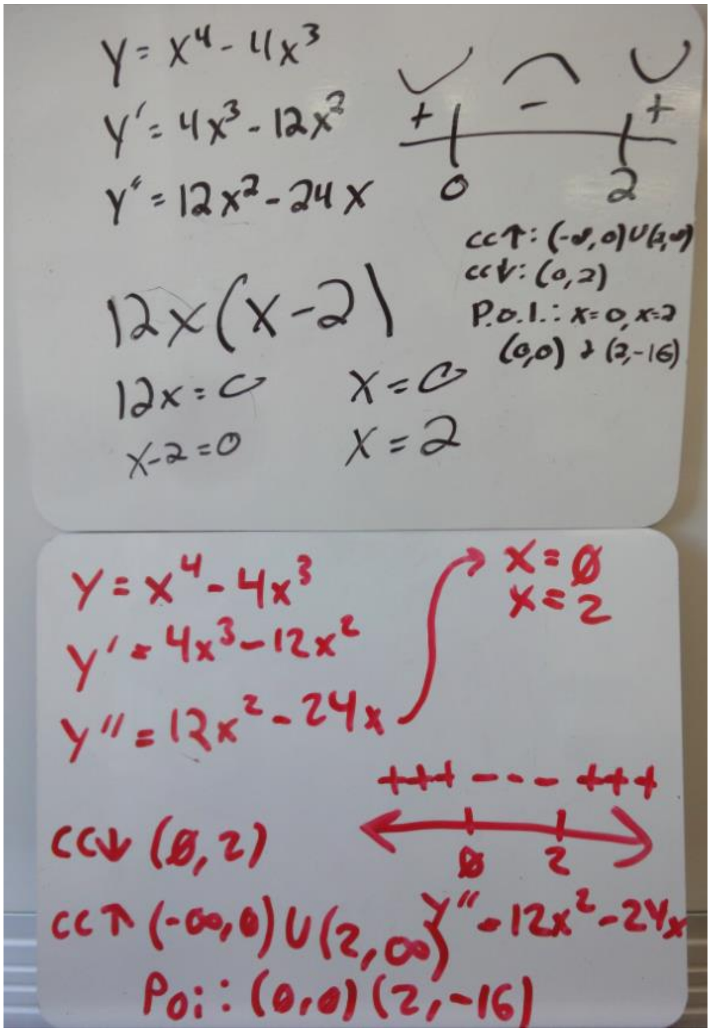
In class we did Problem A, $f(x) = x^3 - 3x$. The second derivative is $f''(x) = 6x$; setting this to zero gives us $x = 0$. Therefore, $x = 0$ is a test value (a potential point of inflection). Using the usual number line analysis for f'', we get this:



The graph of f is concave upward on $(0, \infty)$ and concave downward on $(-\infty, 0)$. We have an inflection point at $(0, f(0)) = (0, 0)$. See the graph:



Two versions of problem (b)…both look good:



PROBLEM

For each of the problems below, do the following:

(a) Identify the critical number(s).

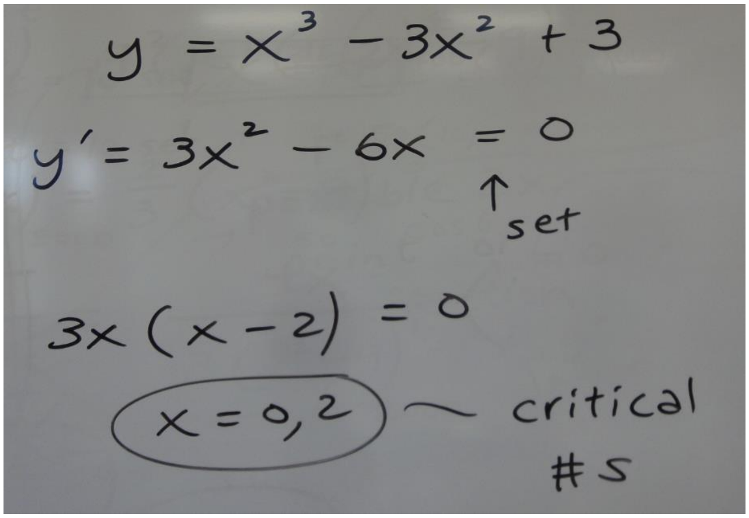
(b) Discuss the concavity of the graph.

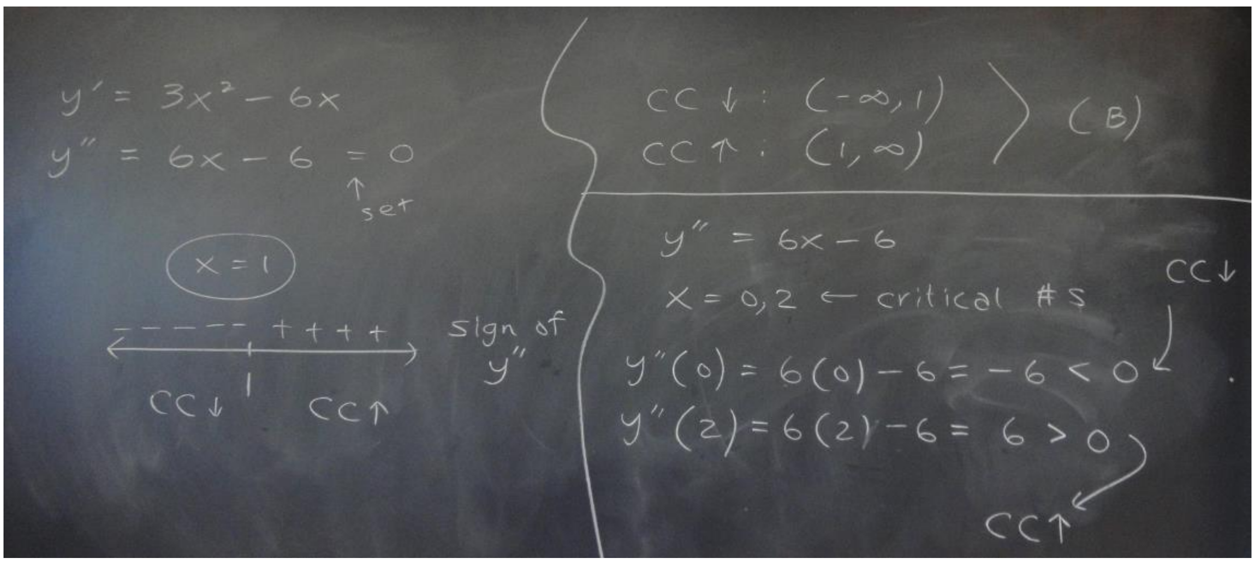
(c) Use the Second Derivative Test to classify the extreme value(s). See part (a).

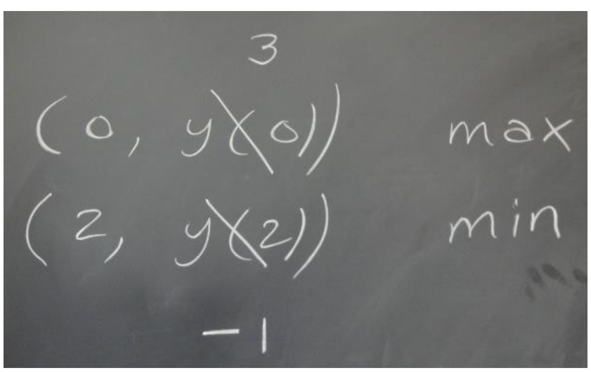
(d) Confirm the above on a calculator.

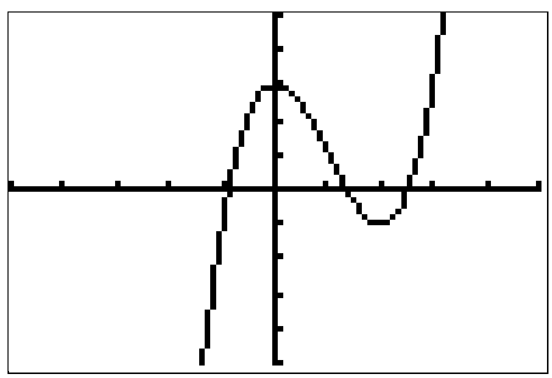
(a) $f\left(x\right) = x^3 - 3x^2 + 3$

(b) $y = x + 1/x$









So here we figure out that $(0,3)$ is a maximum and $(2,-1)$ is a minimum but this is done **by examining the concavity of the graph**.

Problem B:

Nabb's work: