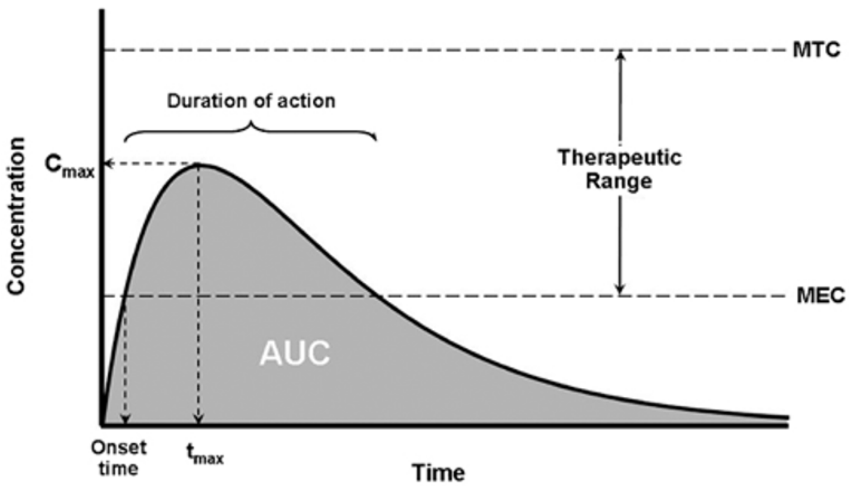
June 6, 2017

3.4: Limits At Infinity; Horizontal Asymptotes

DRUG CONCENTRATION IN THE BLOODSTREAM AS A FUNCTION OF TIME



Source: medscape.com

PROBLEM

Study the following three limits.

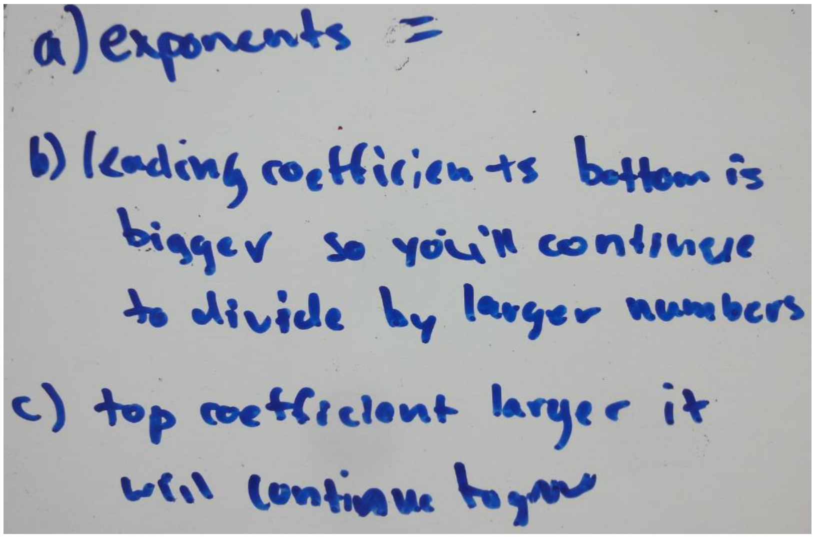
(a) $\lim_{x\to\infty} \frac{2x^2-4}{3x^2+x}$, $L=\frac{2}{3}$

(b) $\lim_{x\to\infty} \frac{2x^2-4}{3x^3+x}$, $L=0$

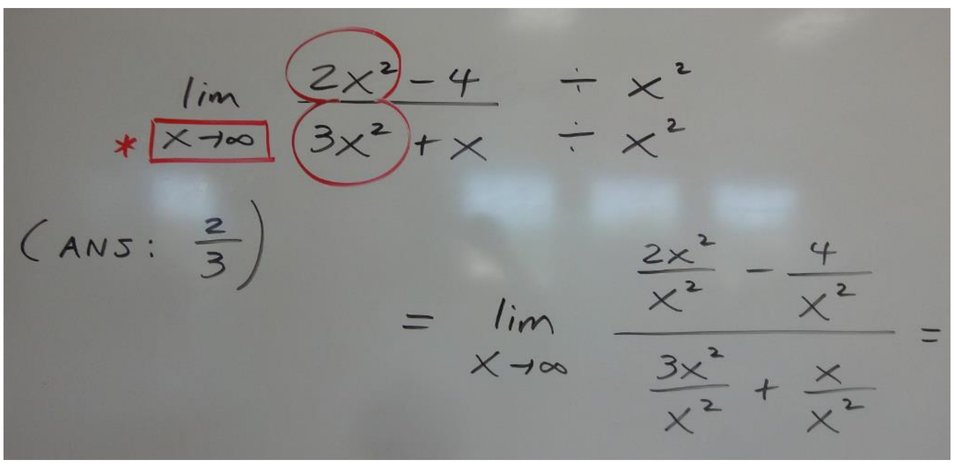
(c) $\lim_{x\to\infty} \frac{2x^3-4}{3x^2+x}$, limit does not exist.

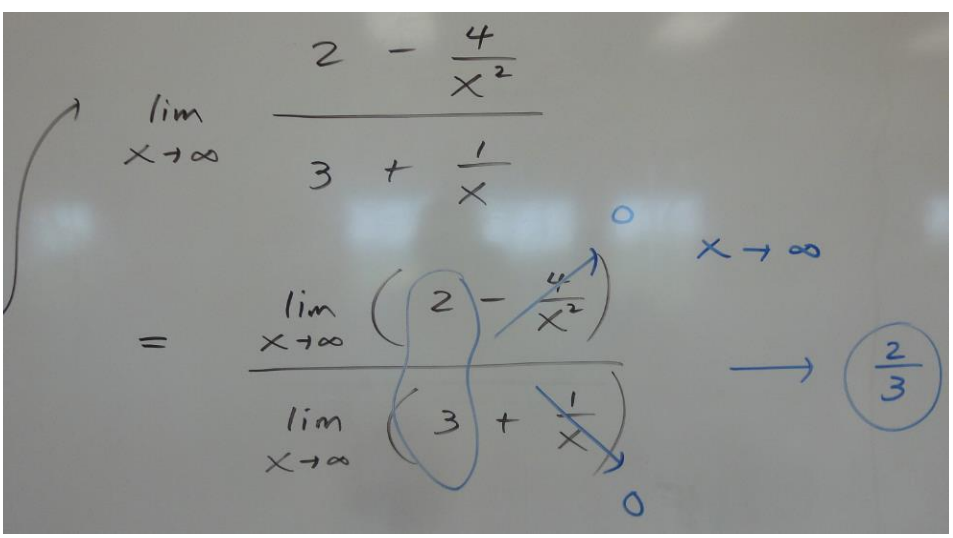
Explain how these limits can be determined by inspection.

Answering these questions informally:



Showing all the details for part (a):

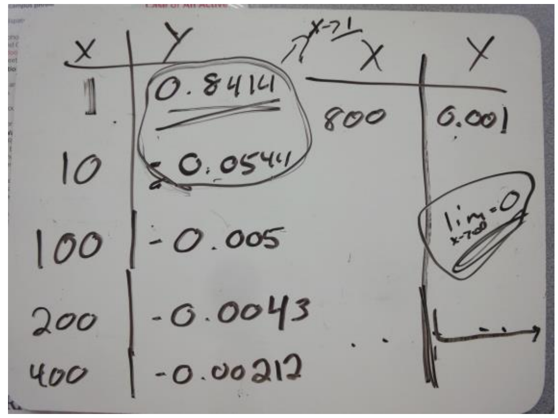




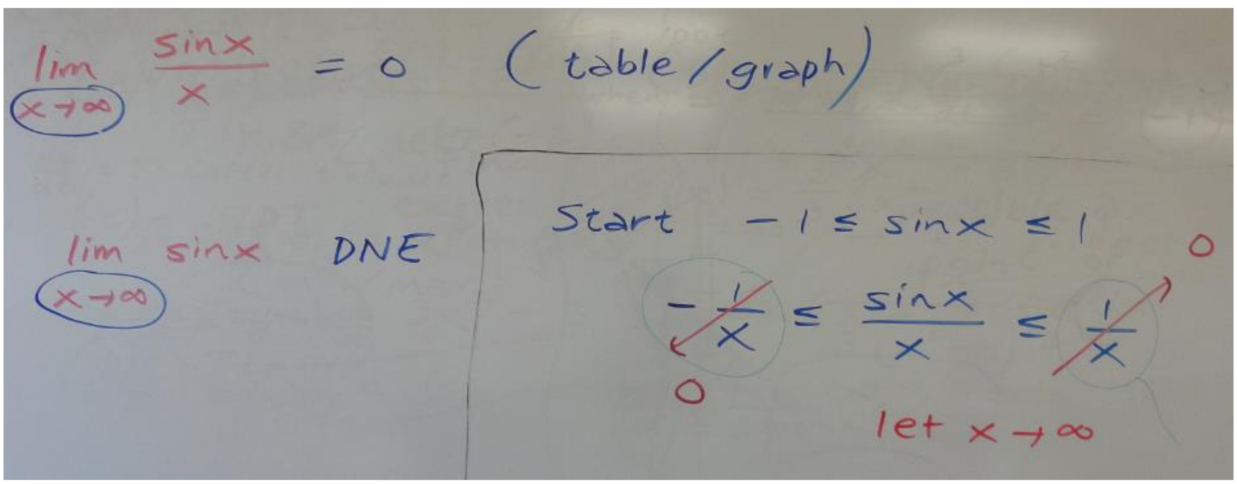
PROBLEM

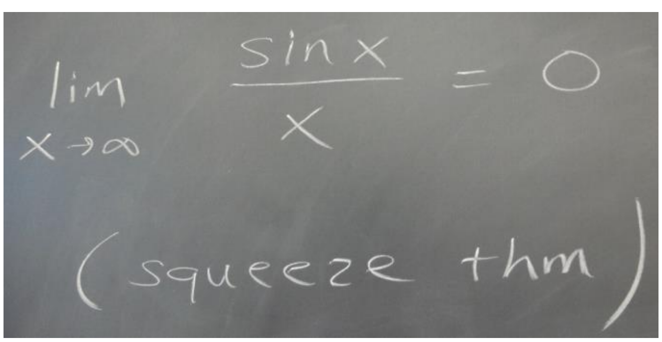
Evaluate $\lim_{x\to\infty}\frac{\sin x}{x}$.

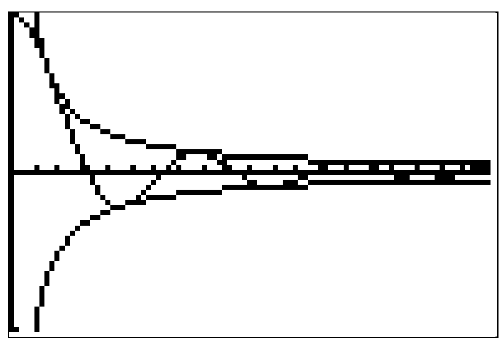
Compare/contrast this with the problem $\lim_{x\to\infty}\sin x$.



A great way to start this problem is to look at a table (see board, upper right). From the table, it appears that $\lim_{x\to\infty}\frac{\sin x}{x}=0$. Notice, on the other hand, that $\lim_{x\to\infty}\sin x$ does not exist. An analytical way to approach this problem is to apply the Squeeze Theorem:







Notice how the graph of $y=\frac{\sin x}{x}$ along with $y=\frac{1}{x}$ and $y=-\frac{1}{x}$ further support this argument.

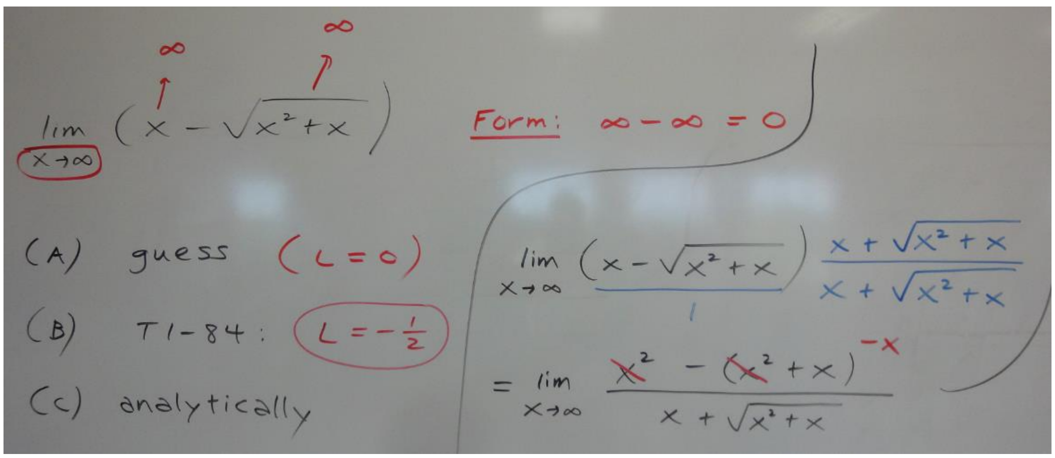
PROBLEM

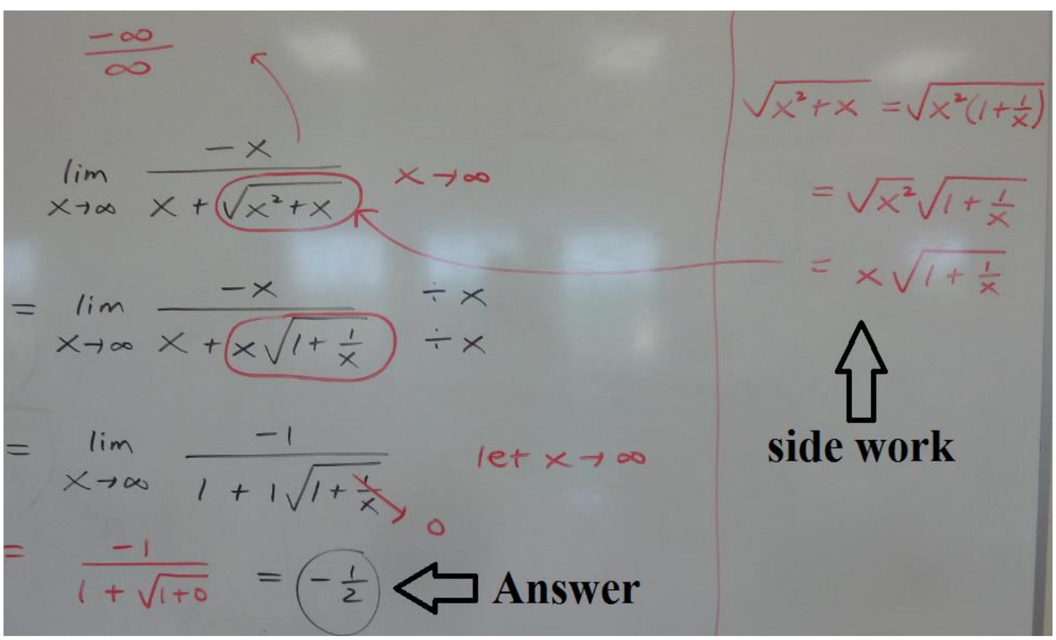
Determining $\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)$.....

(a) What does your intuition tell you about $x-\sqrt{x^2+x}$ for large values of x?

(b) What does a table say?

(c) Find $\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)$ analytically (this is challenging).



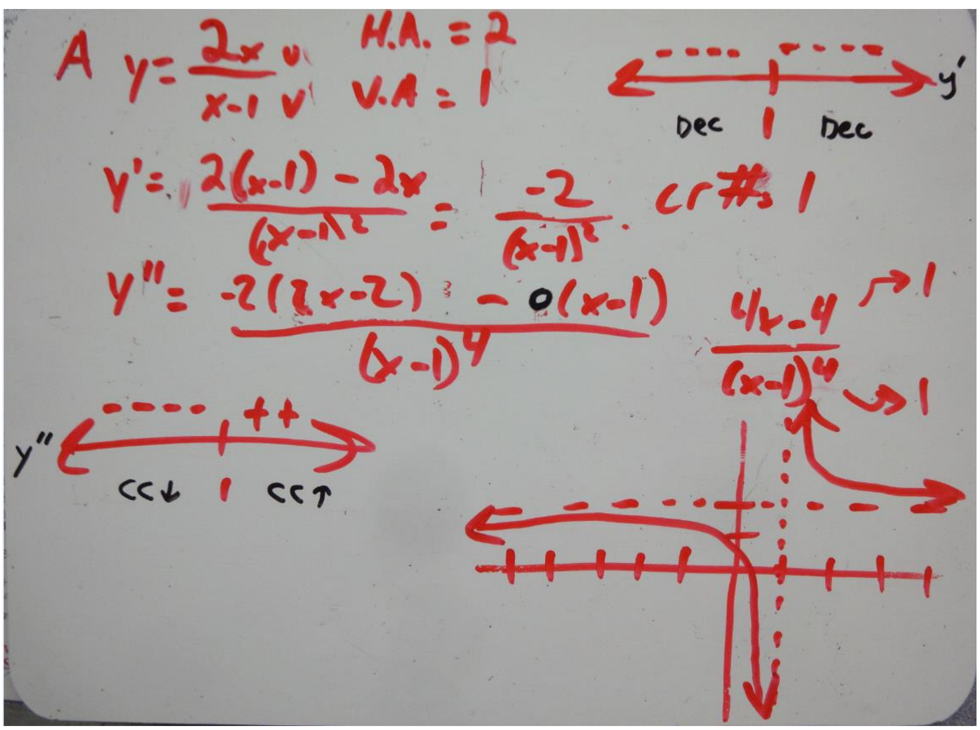


3.5: A Summary of Curve Sketching (Review of previous sections)

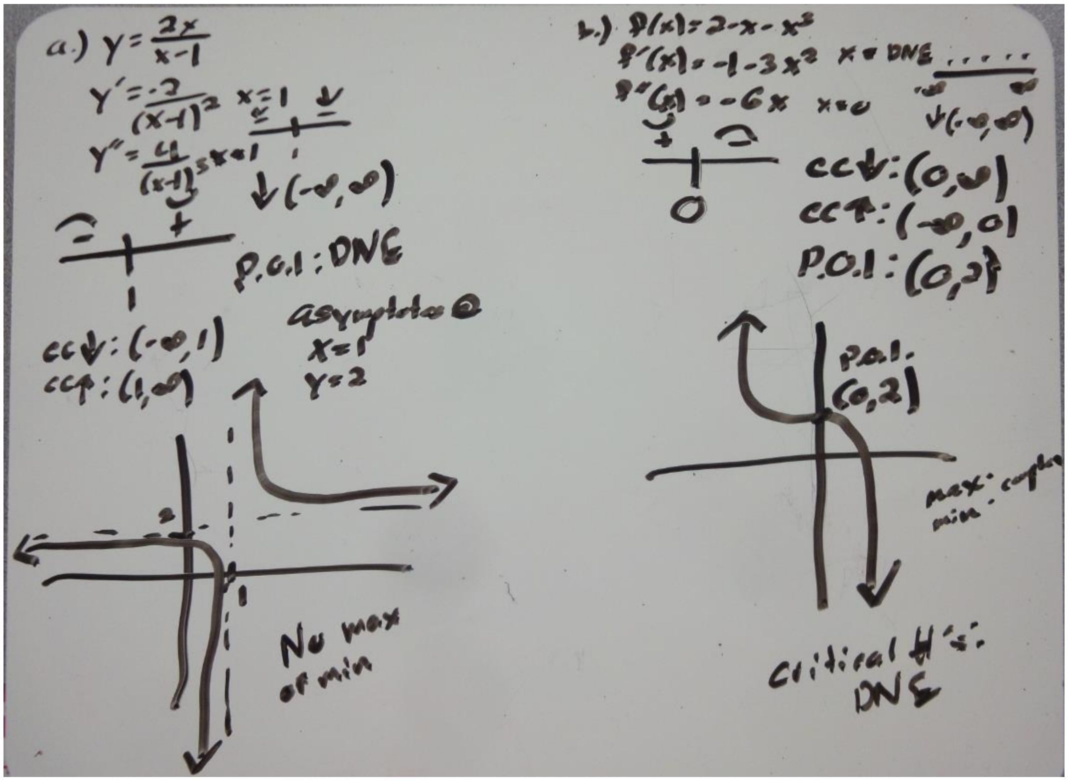
PROBLEM

Carry out a **full analysis** of the function. In other words, study the **first derivative** (intervals of increase/decrease, extreme values), the **second derivative** (concavity, points of inflection), and any other pertinent characteristics (asymptotes, cusps, etc.). Also, provide a freehand sketch based on your findings.

(a) $y = \dfrac{2x}{x-1}$ (b) $f(x) = 2 - x - x^3$



Problem A: If we were being careful here, we would not call $x = 1$ a critical number. Why? It is because although it makes the derivative undefined, it is not in the domain of the function (so it couldn't possibly be a minimum or maximum). Even so, it's a good idea to include this number on the number line analysis for y'. Also, although the graph changes concavity at $x = 1$, this does not imply a point of inflection is there. Why? Because (again) $x = 1$ is not in the domain of the function ($x = 1$ is a vertical asymptote) so it could not correspond to an inflection point.

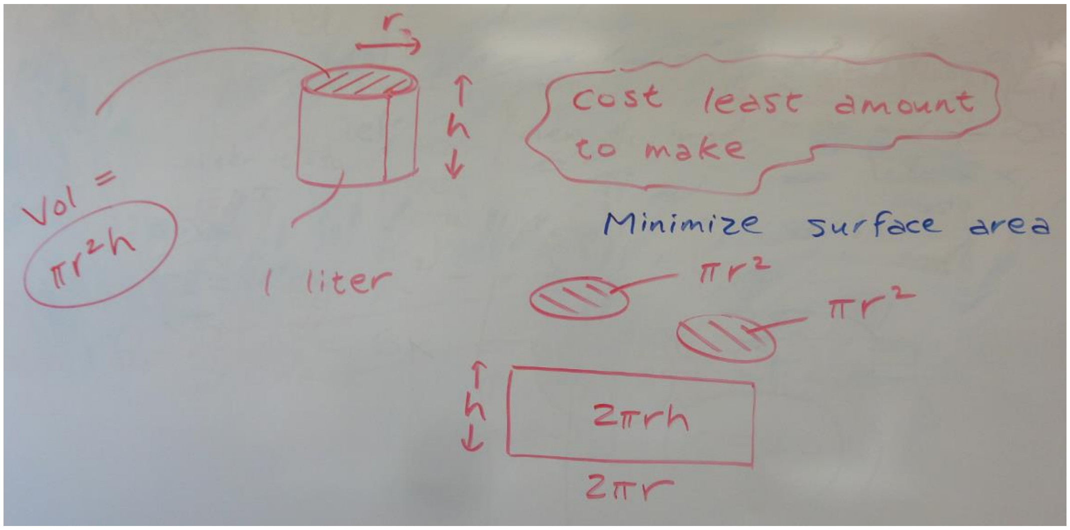


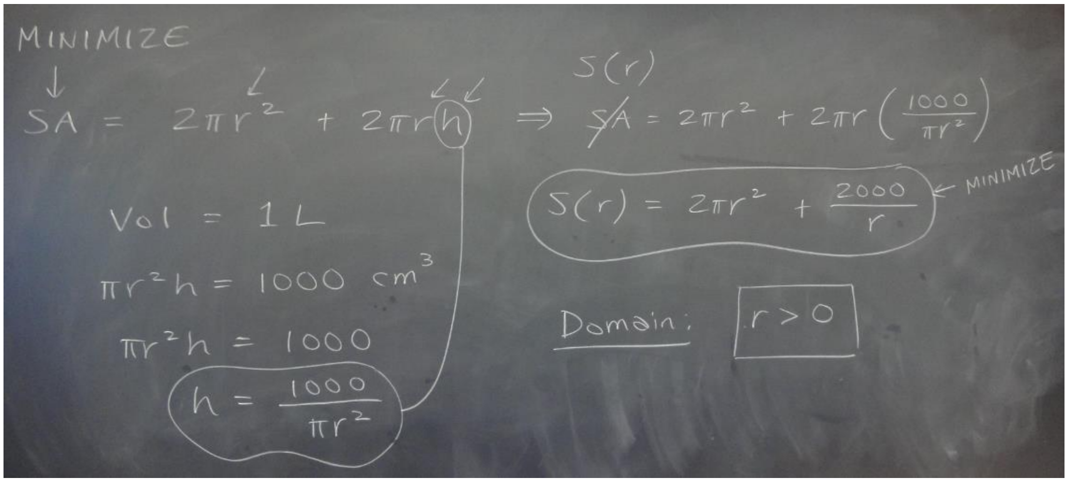
Problem B: An interesting feature here is that this function has no critical numbers. Any number tested in the derivative will be negative, implying that the function is decreasing from $(-\infty, \infty)$.

3.7: Optimization

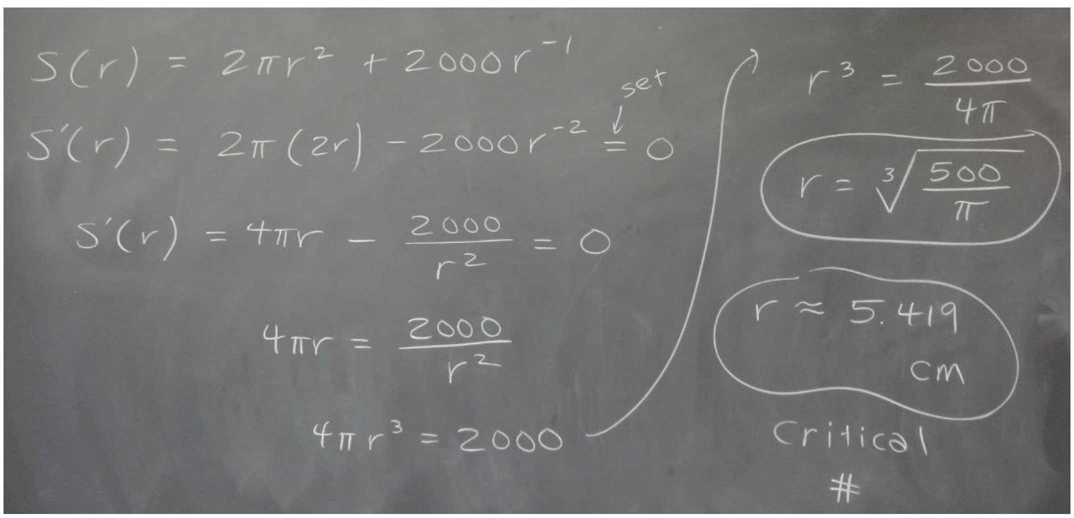
PROBLEM

A cylindrical can holds 1 liter of oil. What are the dimensions of the can that costs the least to manufacture?

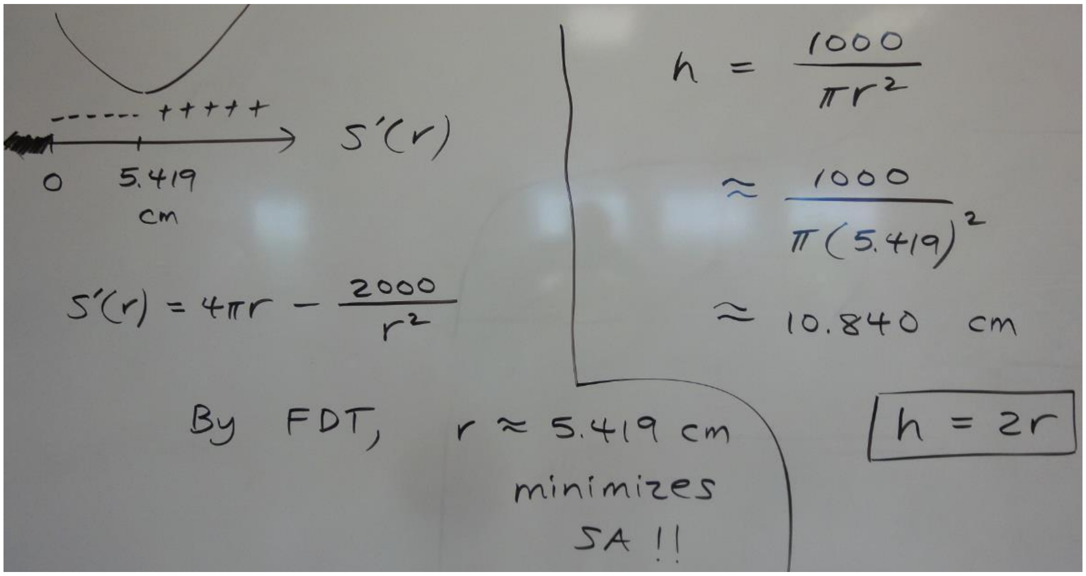




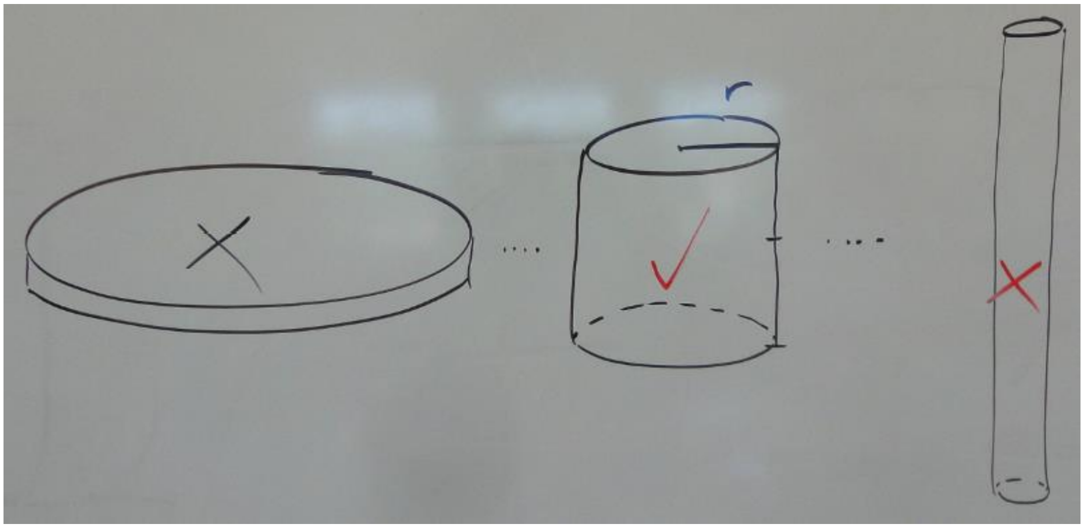
All of the math modeling is done above. We have the equation we need to optimize in terms of **one** variable (*r*).



This is the step where Calculus (the derivative) helps.

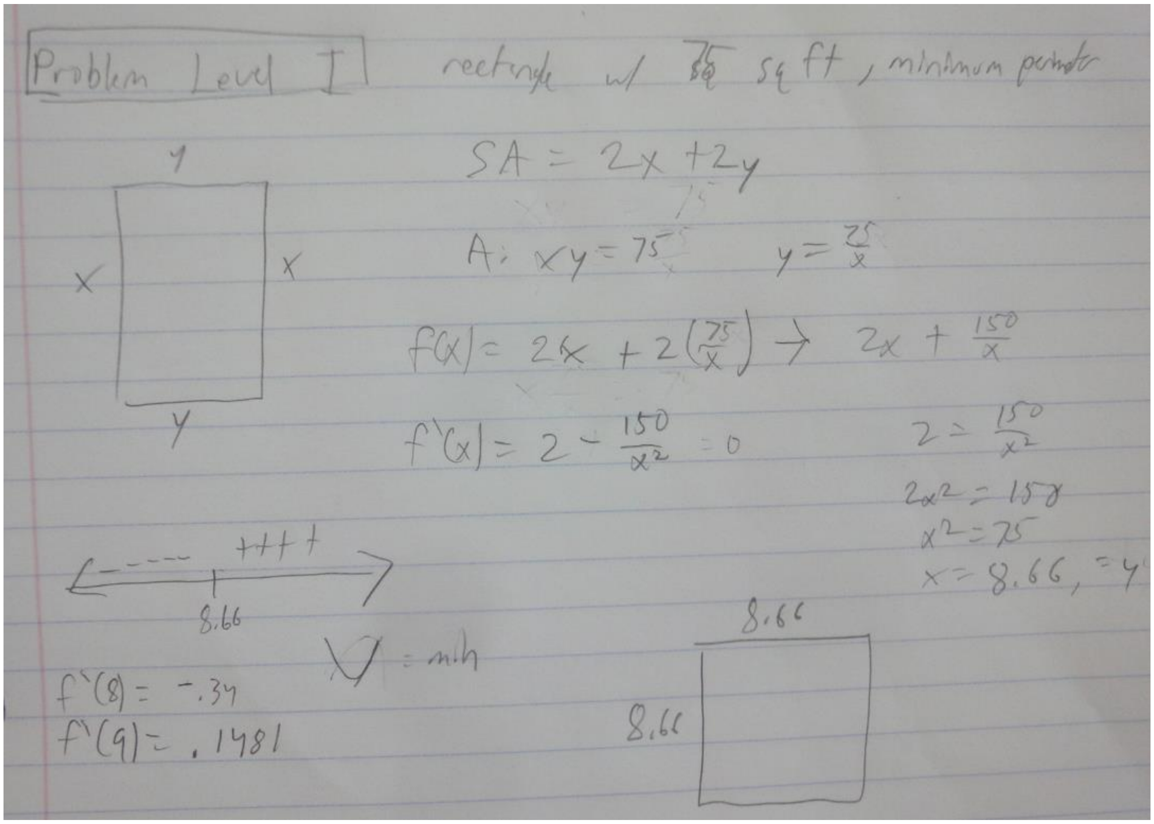


Confirming that $r \approx 5.419$ cm really corresponds to a minimum.

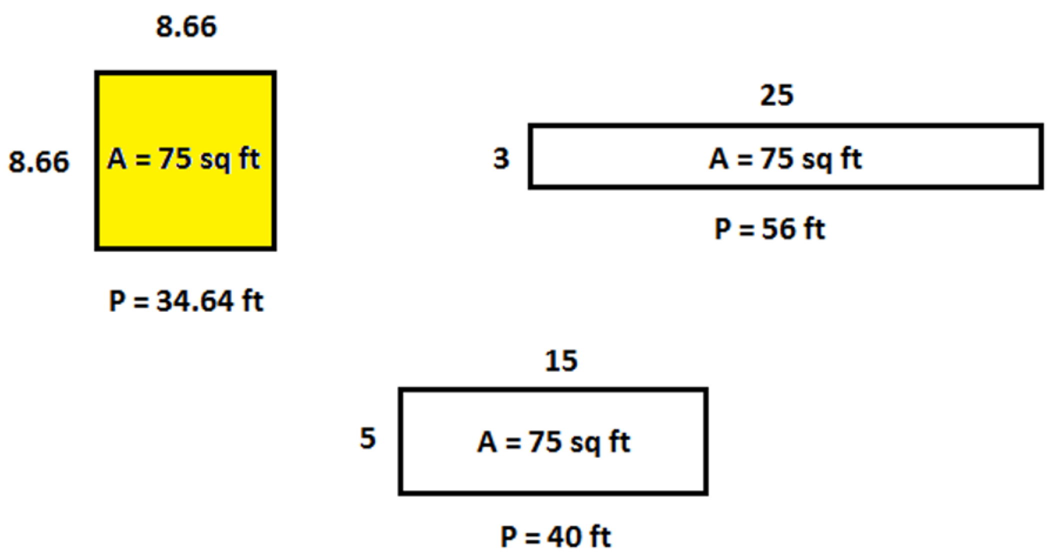


PROBLEM (LEVEL I)

Find the length and width of a rectangle that has area of 75 square feet and a MINIMUM perimeter.



VISUAL SOLUTION



The optimal solution is seen in yellow (it's a square!). The other examples show a rectangle with area 75 sq ft but with perimeters larger than 34.64 ft.