June 7, 2017

3.7 Optimization (continued)

Guidelines that are helpful for many of these problems:

GUIDELINES FOR OPTIMIZATION

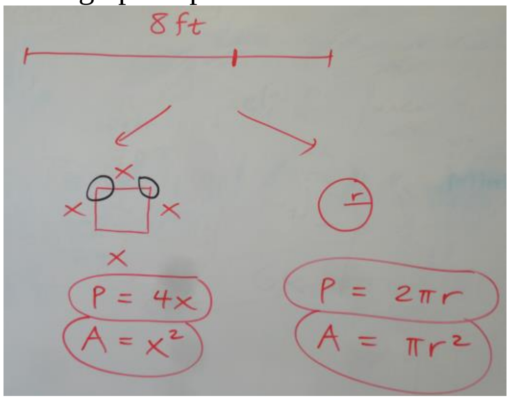
1. **Draw a diagram and assign variables.**
2. **Write down a primary equation for the quantity to be minimized or maximized. (objective function)**
3. **Express the primary equation in terms of one variable by making use of a secondary equation. (constraint)**
4. **Determine the domain of the primary equation.**
5. **Find the desired minimum or maximum.**

PROBLEM (LEVEL II)

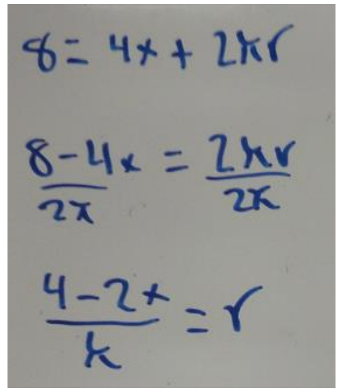
Eight feet of wire is to be used to form a square and a circle. How much of the wire should be used for each figure to enclose the maximum area?

For discussion purposes, use x to denote the side of the square and r for the radius of the circle. Show that the area, as a function of x, is given by $A(x) = x^2 + \pi\left(\frac{4-2x}{\pi}\right)^2$.

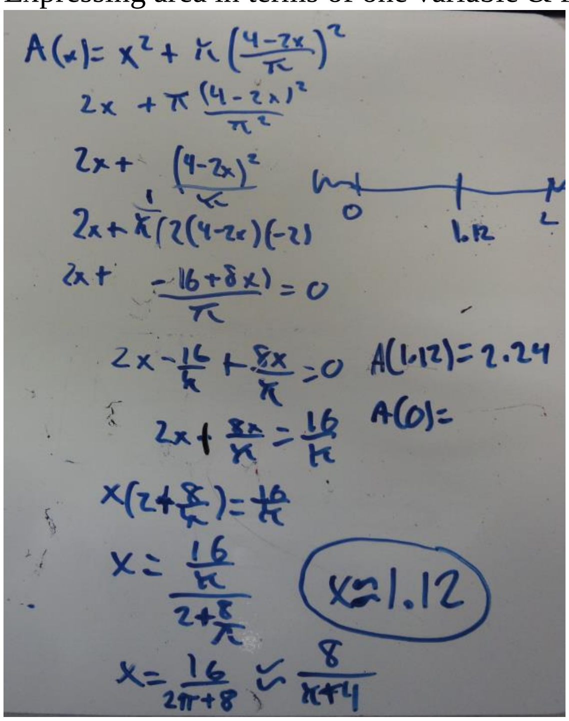
Setting up the problem:



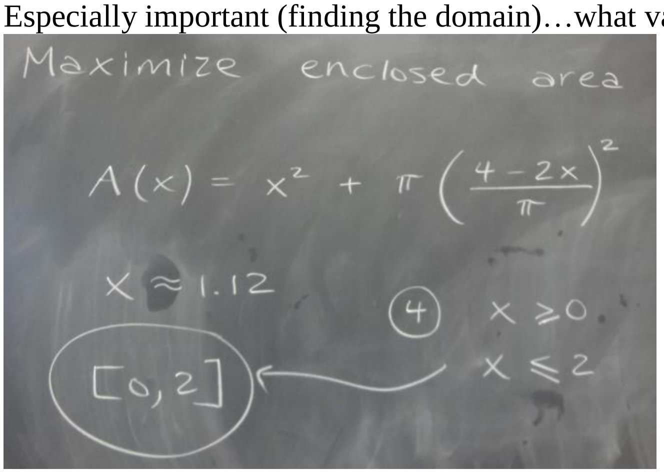
Working with the secondary equation:



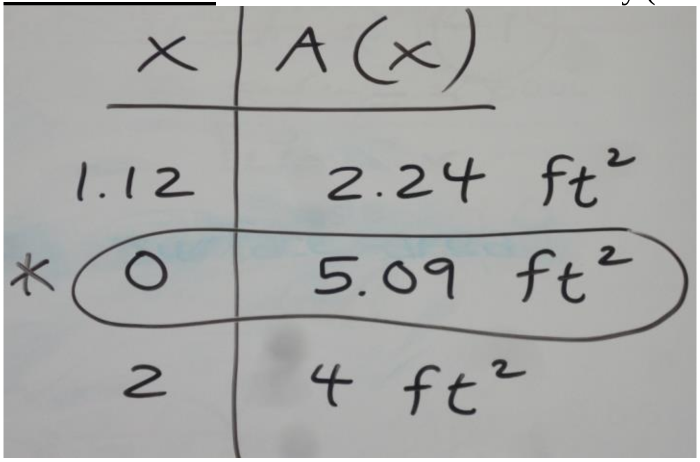
Expressing area in terms of one variable & finding the critical number ($x \approx 1.12$ ft):



Especially important (finding the domain)…what values of *x* make sense in this problem?



Make a table and find the desired maximum. Notice **<u>the maximum does not occur at the critical number</u>** but instead occurs at a boundary (endpoint)!



x	$A(x)$
1.12	2.24 ft^2
* 0	5.09 ft^2
2	4 ft^2

The interpretation here is to make the circle only ($x = 0$). This will maximize the enclosed area!

3.8 Newton's Method

NEWTON'S METHOD

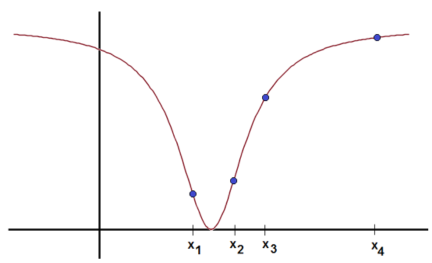
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

$$n = 1, 2, 3, ...$$

$$x_1 : \text{ initial guess}$$

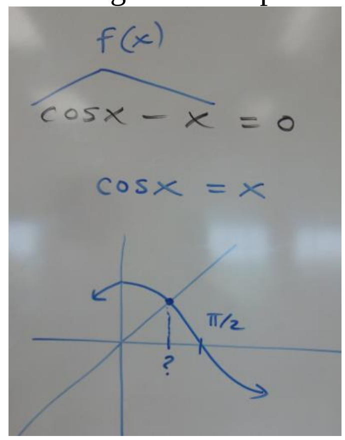
WARM UP

Consider using each of the values below as a starting point for Newton's method. For which of them do you expect Newton's method to "work" and lead to the root of the function?



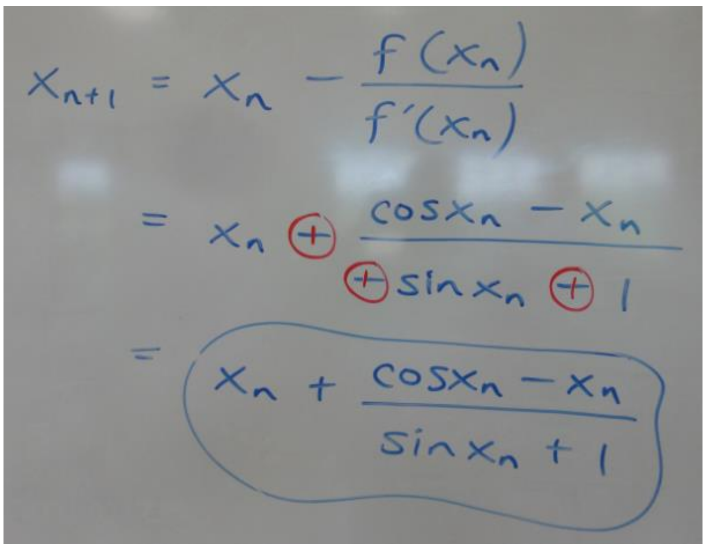
As for the diagram on the right, it seems that choosing x_1, x_2, or x_3 would all lead to the eventual zero (*x*-intercept). On the other hand, it seems that choosing x_4 would lead you *away* from finding the *x*-intercept. The initial tangent line at x_4 is very flat (nearly horizontal). This makes the subsequent tangent line seem even flatter (moving its *x*-intercept farther from our desired target).

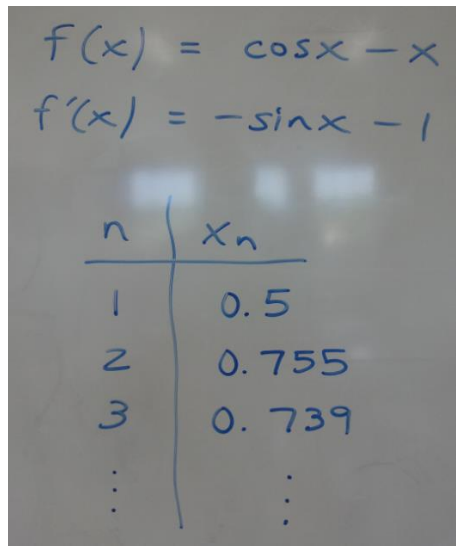
Thinking about the problem:

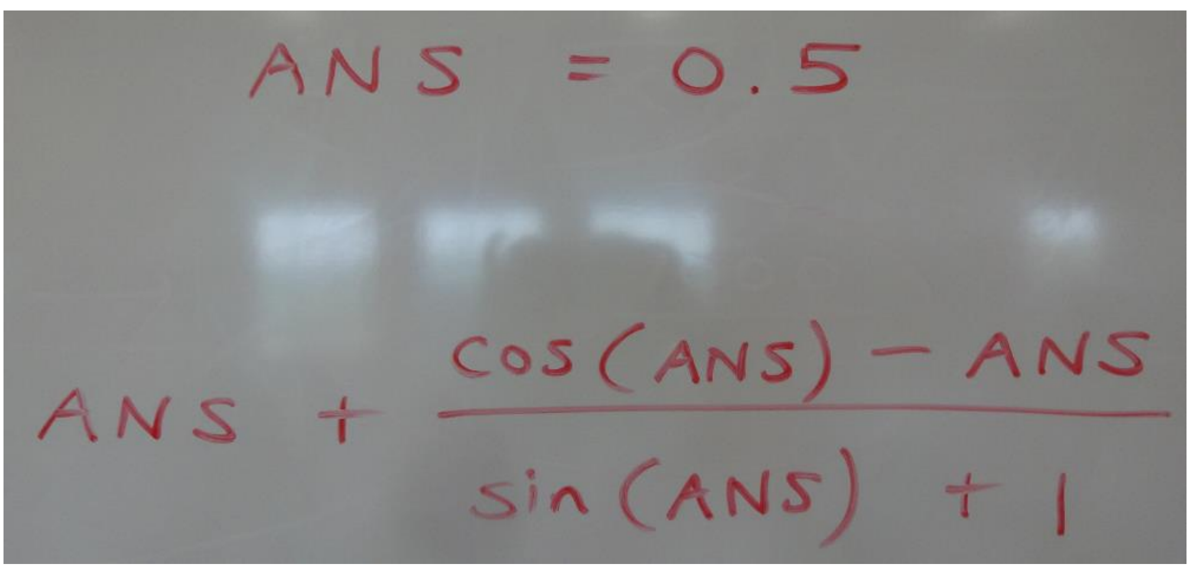


PROBLEM

Approximate the solution to the equation $\cos x - x = 0$.

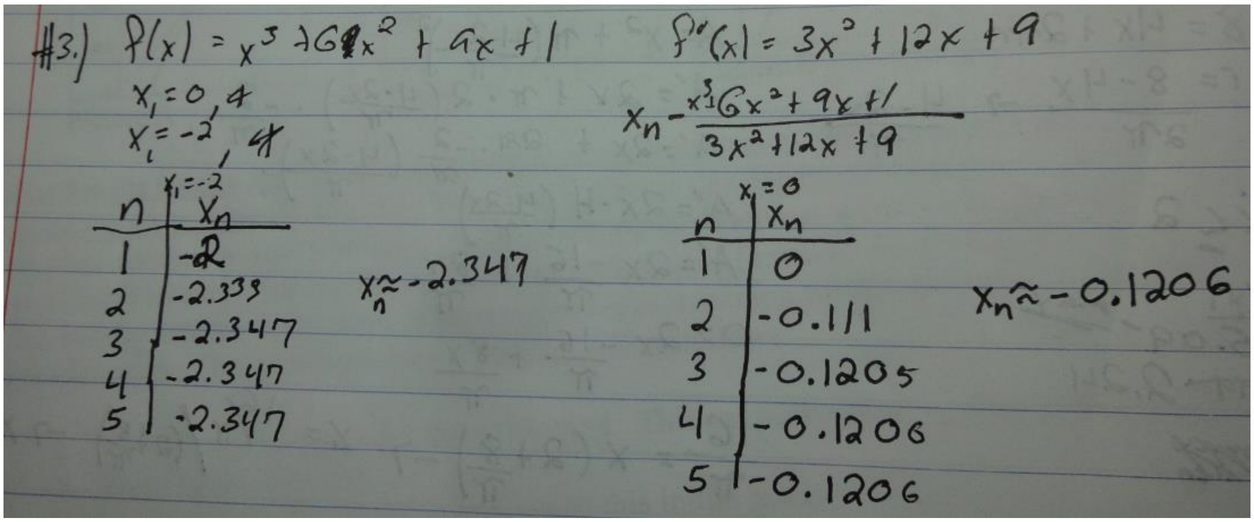






PROBLEM

Approximate the x-intercept to the function $f(x) = x^3 + 6x^2 + 9x + 1$. Use $x_1 = 0$ on one attempt; Use $x_1 = -2$ on another.



The two different answers suggest the function passes through the x-axis (at least) twice—once at around $x \approx -2.347$ and once around $x \approx -0.1206$.

3.9 Antiderivatives

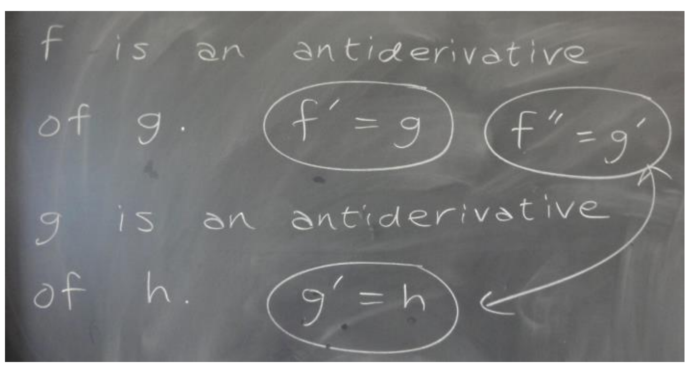
WARM UP

If f is an antiderivative of g, and g is an antiderivative of h, then

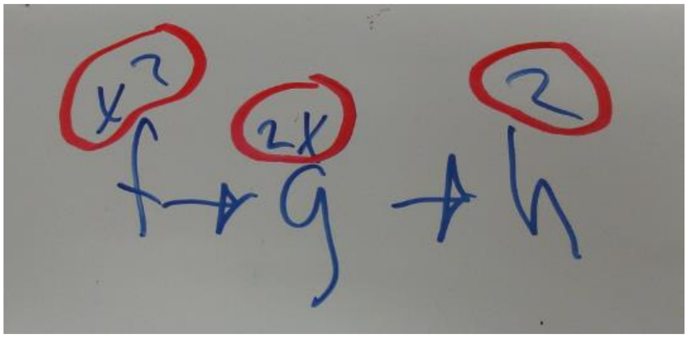
(a) h is an antiderivative of f.

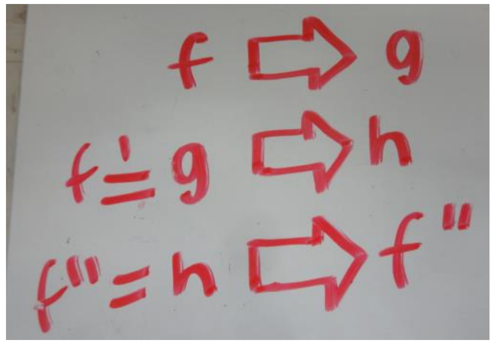
(b) h is the second derivative of f.

(c) h is the derivative of f''.



Two great solutions:





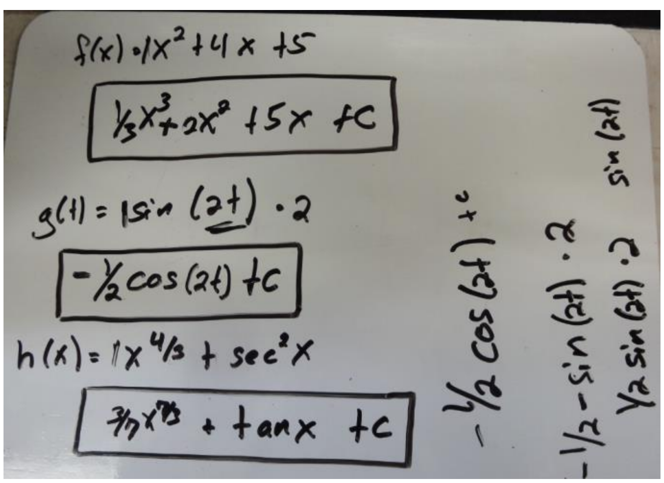
From each of the above, we can see that differentiating f twice gives *h*. Choose (b).

FIND THE ANTIDERIVATIVES:

(a) $f(x) = x^2 + 4x + 5$

(b) $g(t) = \sin(2t)$

(c) $h(x) = x^{4/3} + \sec^2 x$



Answers are in the $\boxed{\text{boxes}}$ above. We can notate them as $F(x)$, $G(t)$, and $H(x)$, respectively.