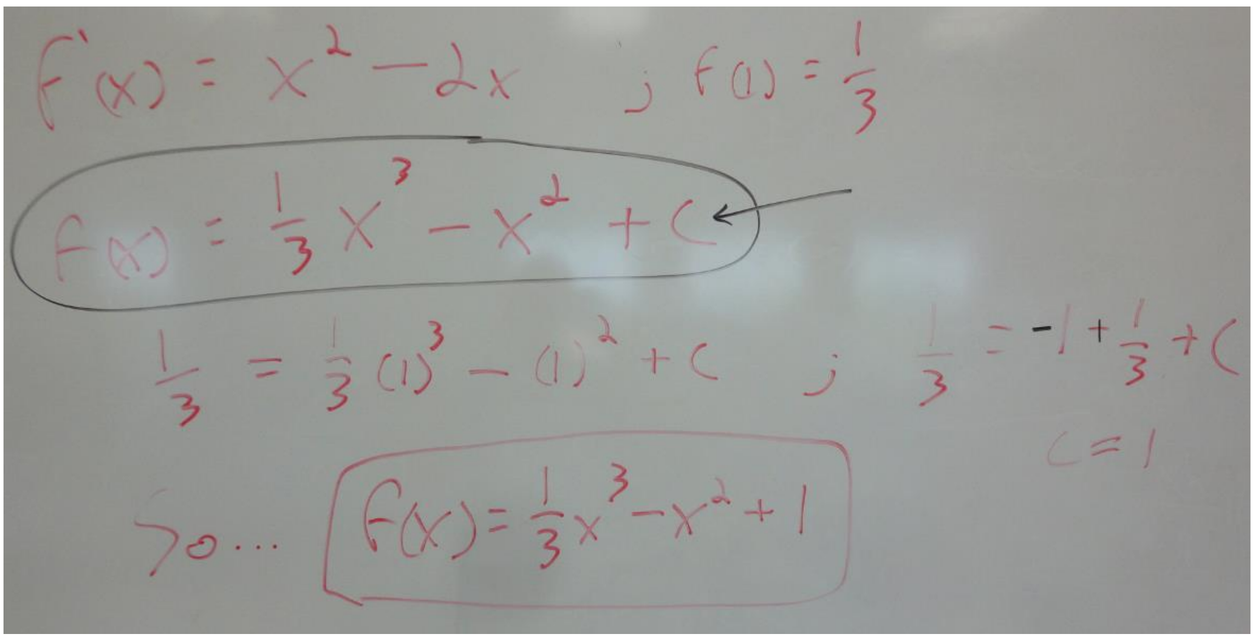
June 8, 2017

**3.9: Antiderivatives (continued)**

**PROBLEM**

Find the unique function $f(x)$ such that

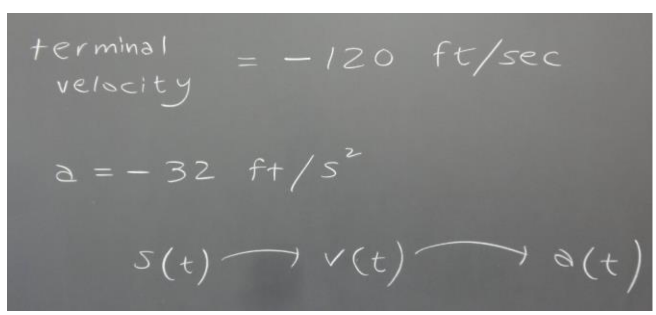
$$f'(x) = x^2 - 2x \text{ and } f(1) = \frac{1}{3}.$$



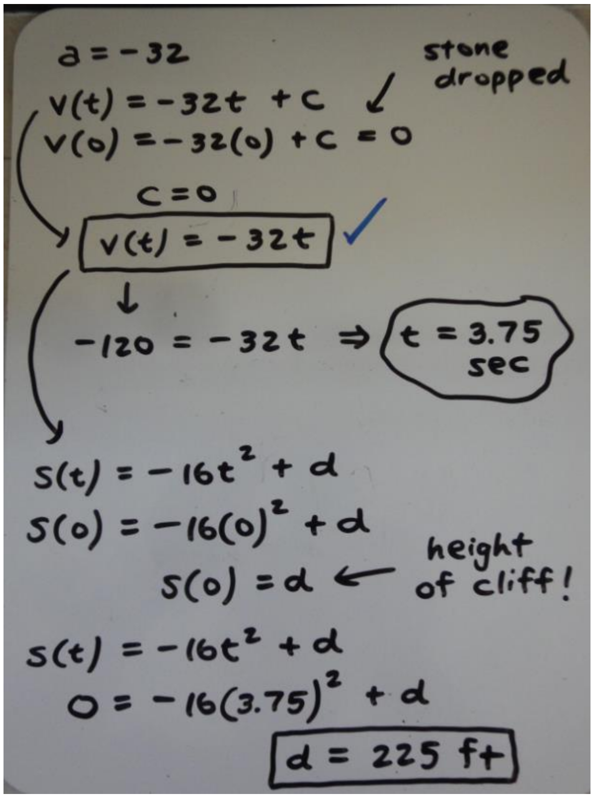

**PROBLEM**

Suppose a stone is dropped from a cliff. The stone hits the water at 120 feet/sec. What is the height of the cliff?

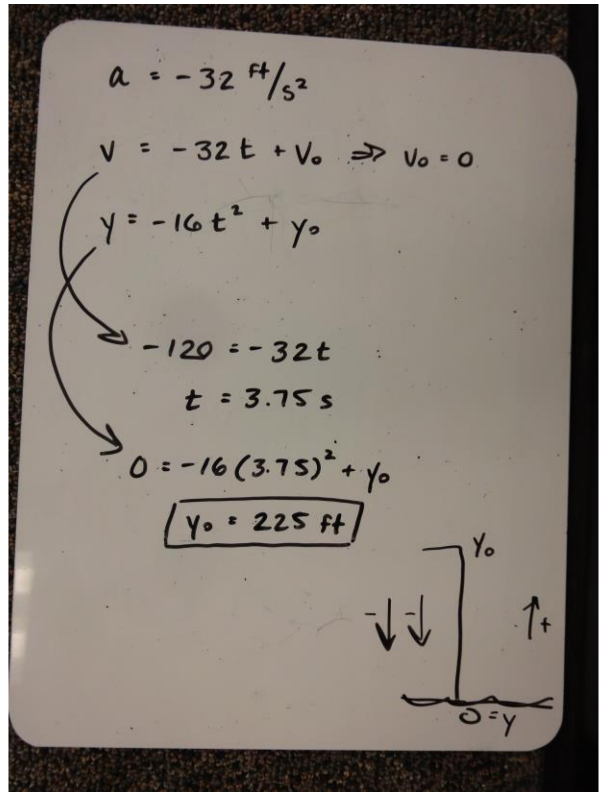
Source: jonas.ph



Nabb's solution:



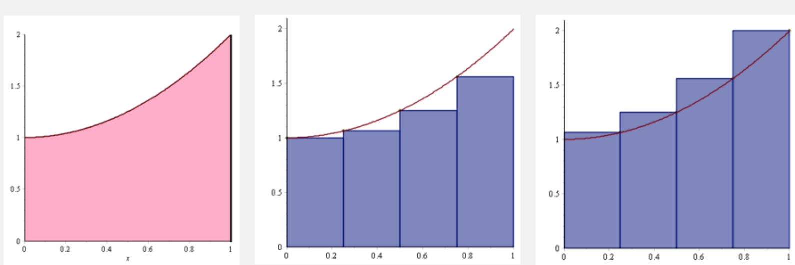

Another nice solution:



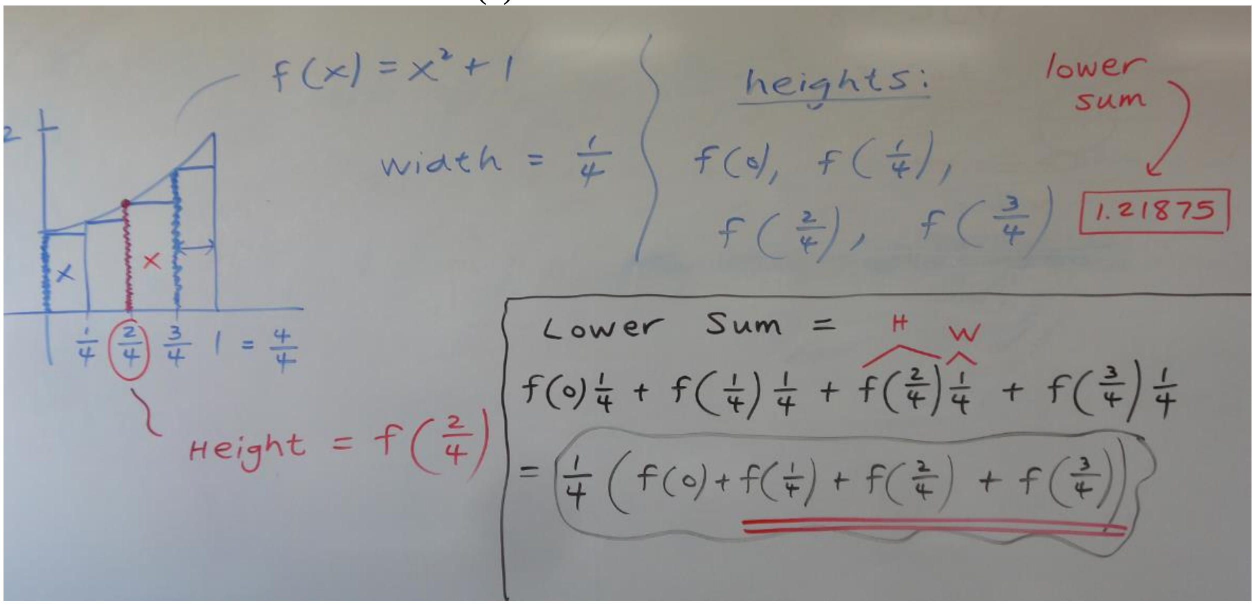

**4.1: Area & Distance (Introduction)**

**PROBLEM**

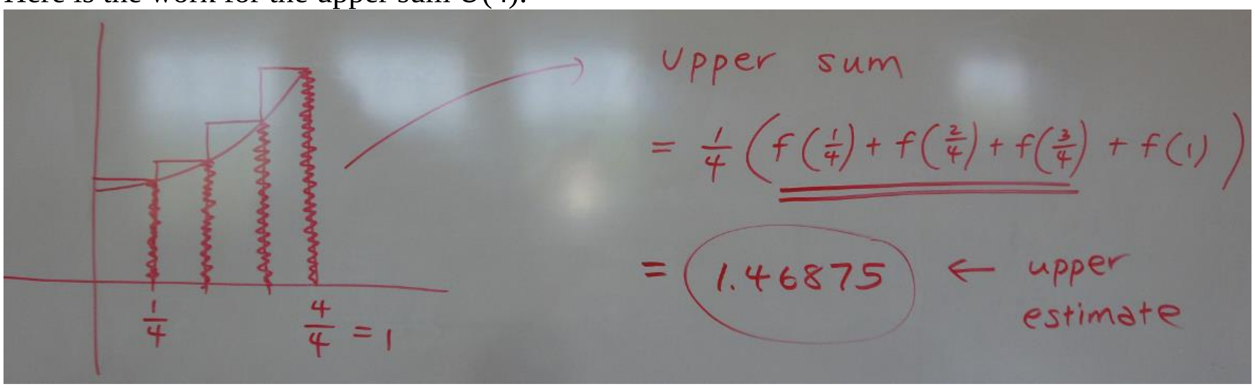
Consider the region bounded by the parabola $f(x) = x^2 + 1$, the $x$-axis, and the vertical lines $x = 0$ and $x = 1$. Let's estimate this area using simple objects: rectangles.



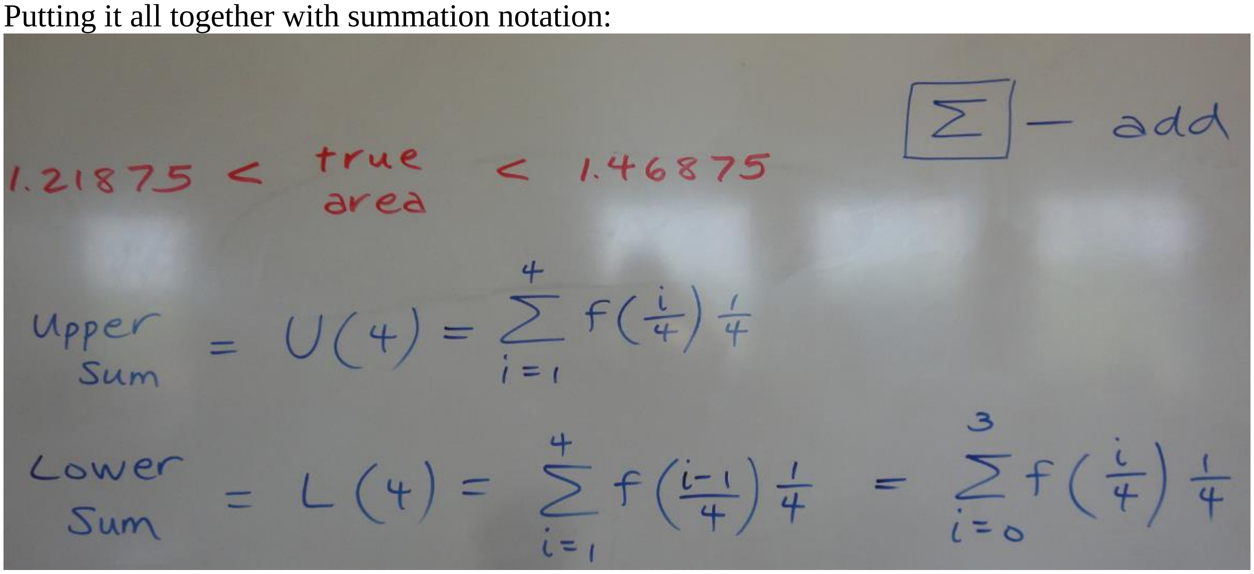
Here is the work for the lower sum $L(4)$:




Here is the work for the upper sum $U(4)$:




Putting it all together with summation notation:



As we can see in the table below, the lower sum continues to *increase* as the number of rectangles increases. The upper sum continues to *decrease* as the number of rectangle increases. Both are tending toward the same number (the exact area under the curve).

## OBSERVATION

Approximate areas (both underestimates and overestimates) using $n$ rectangles.

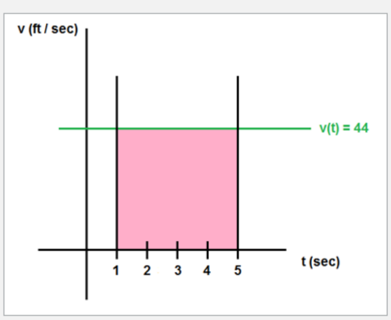
| n | Lower Sum | Upper Sum |
|---|---|---|
| 4 | 1.2188 | 1.4688 |
| 8 | 1.2734 | 1.3984 |
| 20 | 1.3088 | 1.3588 |
| 50 | 1.3234 | 1.3434 |
| 100 | 1.3284 | 1.3384 |
| 500 | 1.3323 | 1.3343 |

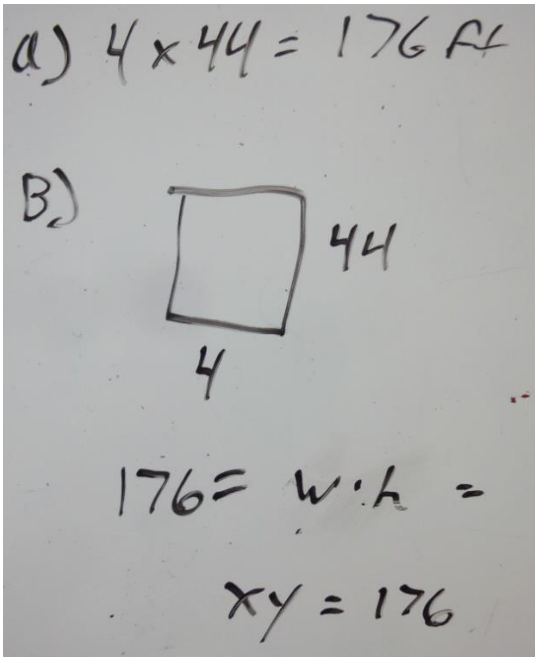
http://www.shodor.org/interactivate/activities/Integrate/

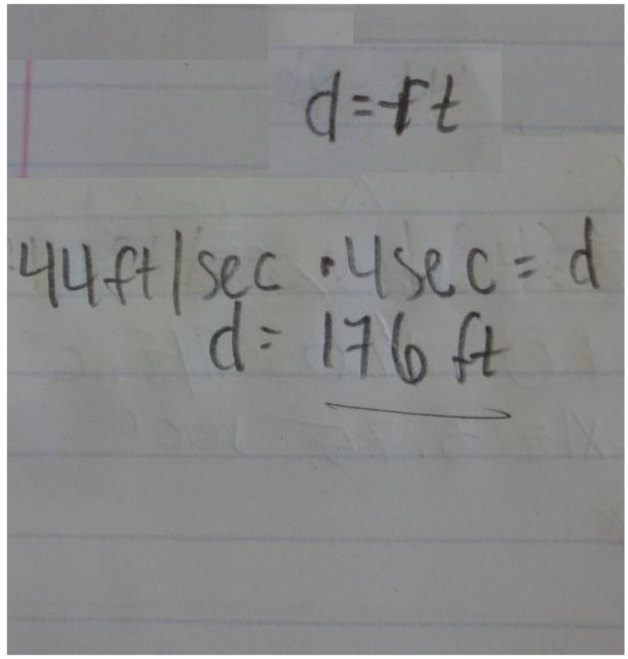
## QUESTION

(a) Consider a car moving along a straight road with constant velocity $v(t) = 44$ ft/sec. What is the distance traveled by the car from $t = 1$ sec to $t = 5$ sec?

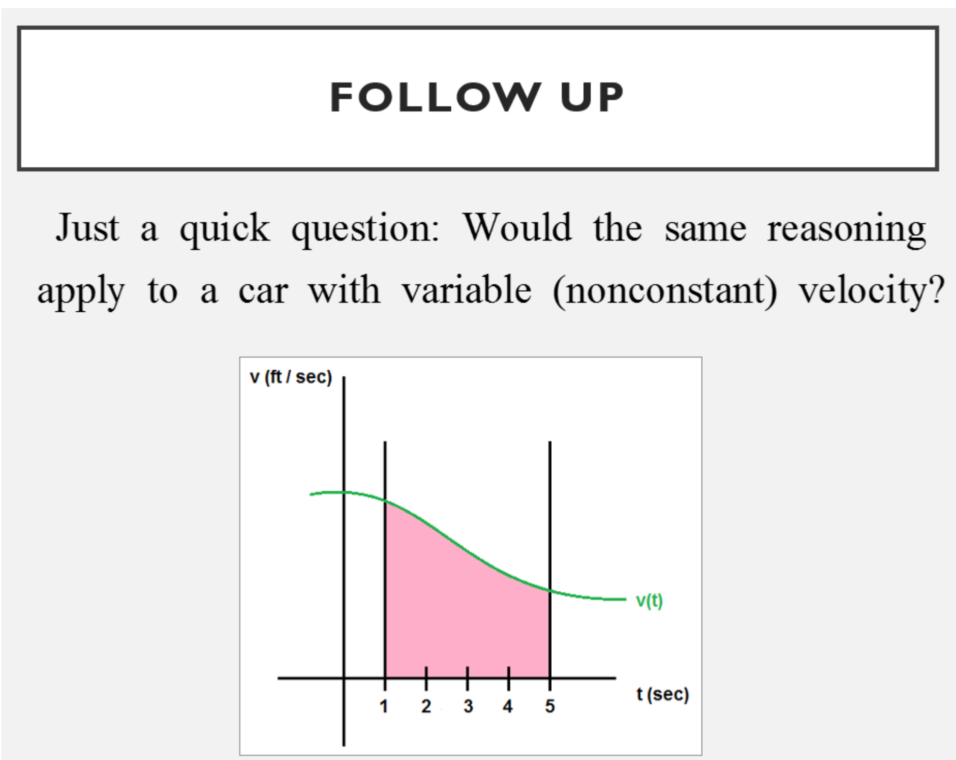
(b) What geometric calculation gives the answer (instantly!) to the above question? See the figure.






In this context, displacement/distance is equivalent to the area under the curve.



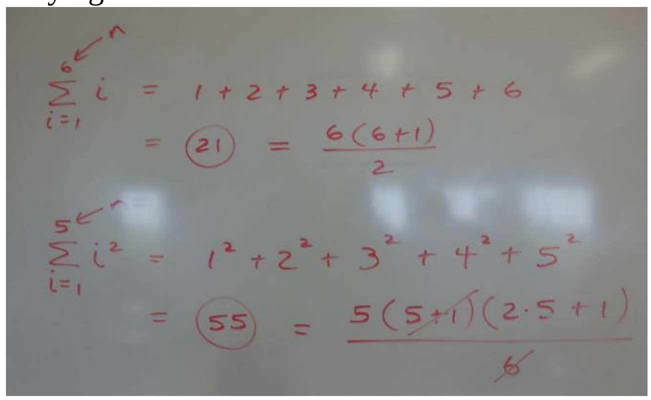

ABSOLUTELY!!!

# 4.2: The Definite Integral

**SUMMATION FORMULAS (PAGE 309)**

1. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
2. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
3. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

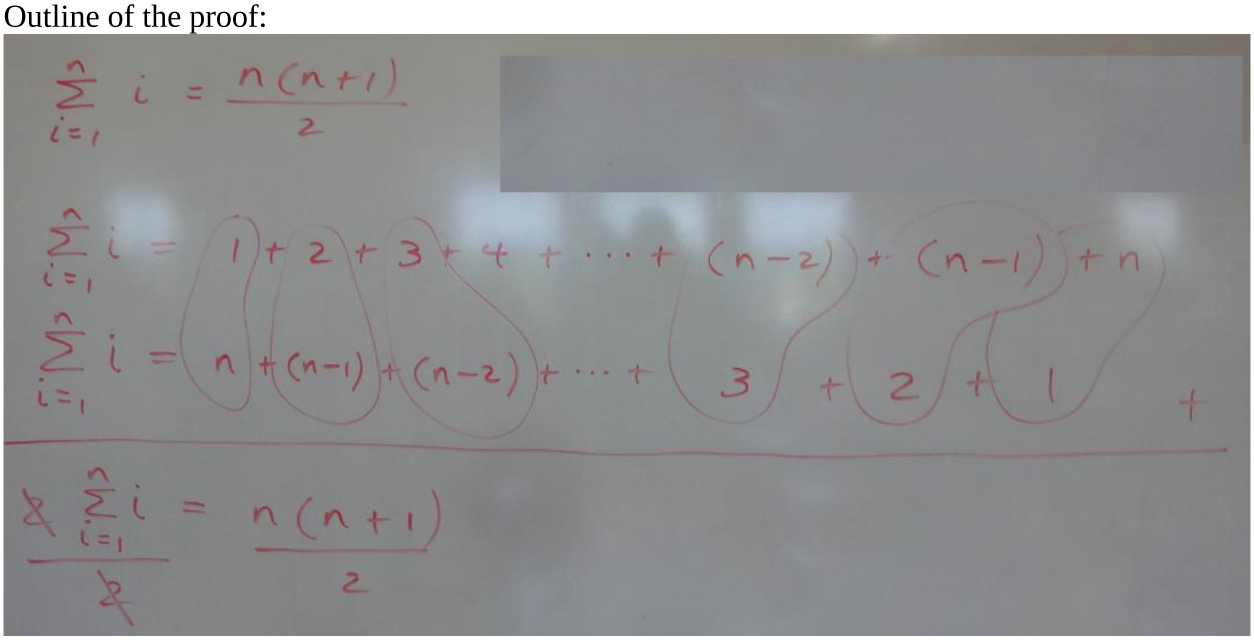
Playing around with summation:



**EXERCISE**

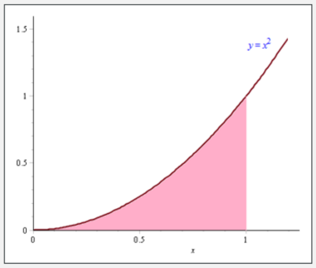
Can you prove that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$?

Outline of the proof:

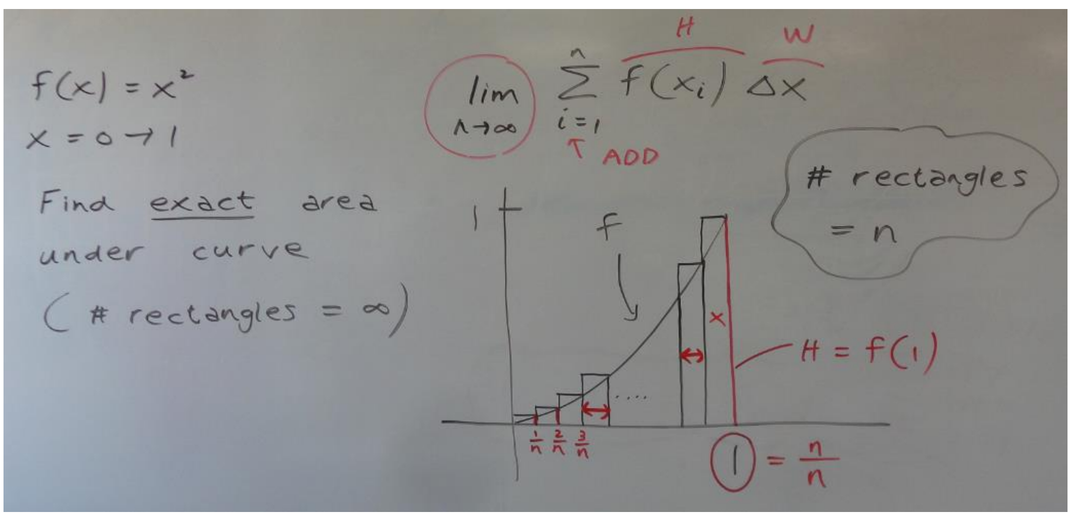


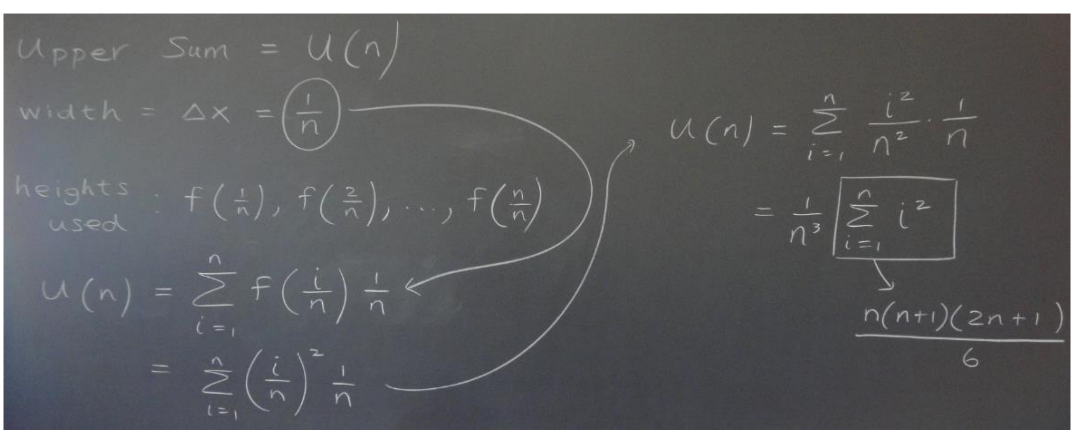
**PROBLEM**

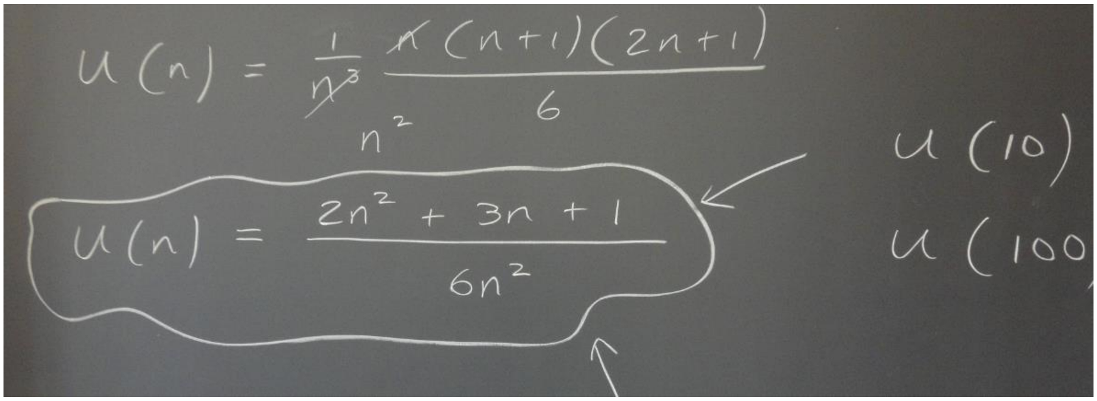
Consider the region bounded by $f(x) = x^2$, $x = 0$, $x = 1$, and the $x$-axis:

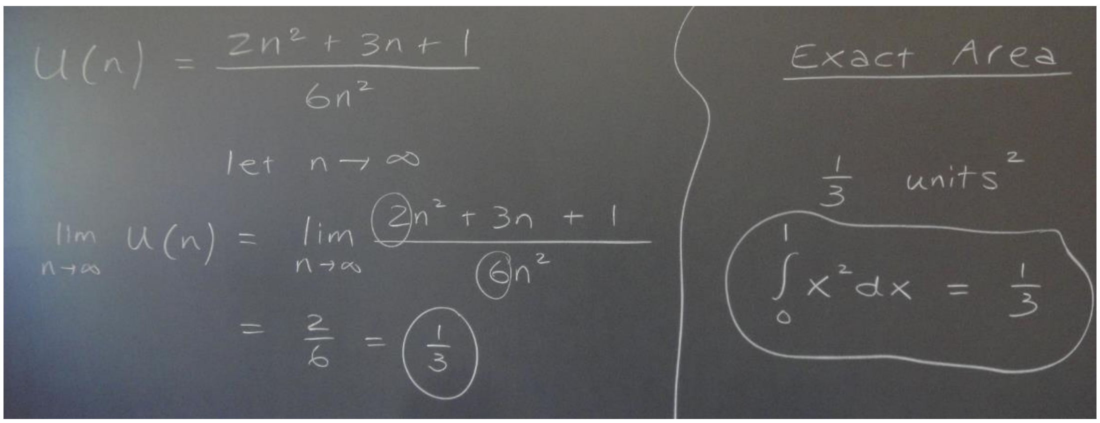



Use an infinite number of rectangles to determine this area **exactly**.

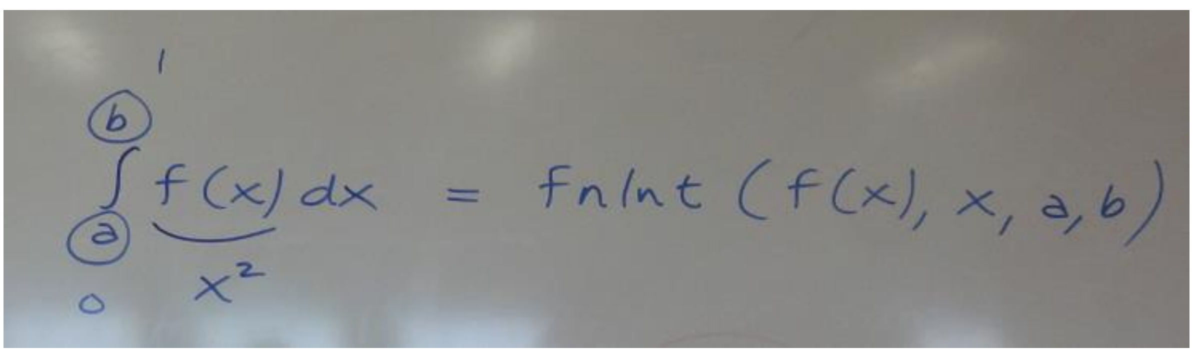


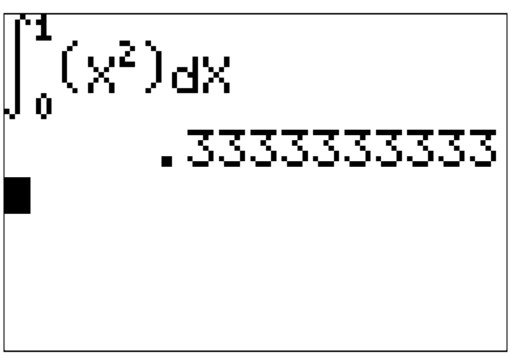






Doing this on the calculator:




or