MATH 126/247 **Name:** Key

M&M Activity

Data, Regression, R^2, Residuals

Please find a partner.

You will need:	1. A bag of M&Ms 2. A small paper cup 3. Paper towel 4. Desmos (free online math program)

Part I: Watch the Function Grow

1. Start by putting 4 M&Ms in a cup. We will call this

(trial number, number of M&Ms) = (0, 4).

2. Shake the cup and pour the M&Ms out onto the paper towel. Count the number of M&Ms that have the M showing. Add an M&M for each one with an M showing. For example, if two Ms were showing, you'd now have 6 M&Ms so this would be the point

(trial number, number of M&Ms) = (1, 6).

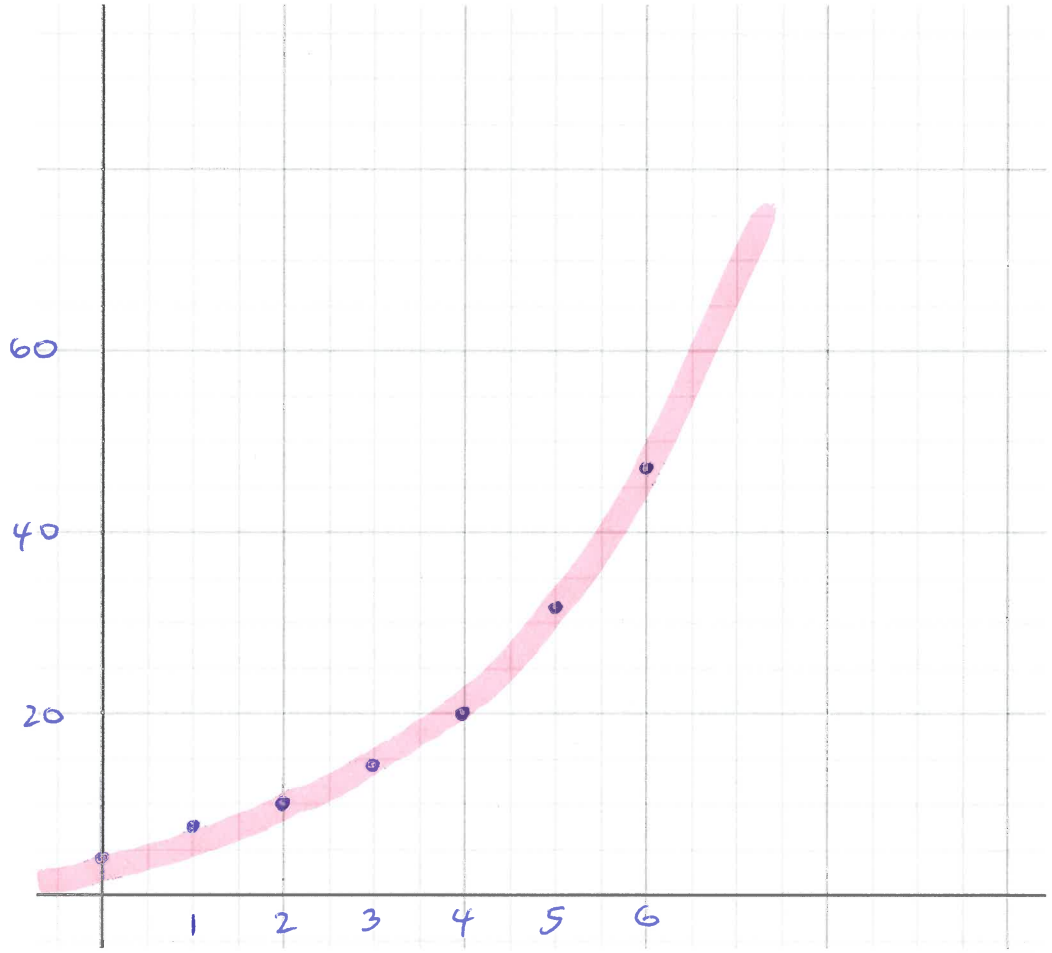
3. Repeat for 6-7 trials. Enter your data in Desmos (see step 4 below).

4. Visit Desmos.com. You can put your data into a table (see the + button). The x_1 is the trial number while the y_1 is the number of M&Ms in the cup just before the next trial begins.

5. Once all of your data is in the table, enter the expression $y_1 \sim ab^{x_1}$ in a new command line. (Note: When you type y1, Desmos will automatically recognize this as y_1.) The function (curve) you are building is called a **regression model**. What is the regression model attempting to do?

It is attempting to describe the trend in the data (and is good for making projections / predictions).

6. You should change the viewing window so you can see all of the data and the regression model. The button will allow you to change the x range and the y range. Something like $-3 \le x \le 8$ and $0 \le y \le 70$ may work—it depends on how many trials you conducted and what your data look like. Reproduce the data points and the regression curve on the next page. Choose an appropriate scale on the x and y axes.



X_1	Y_1
0	4
1	7
2	10
3	14
4	20
5	32
6	47

Model: $y = 4.24(1.49)^x$

7. Play around a bit.

(a) For example, Desmos gives the values of a and b. What do you think these values mean in the context of the M&Ms?

$a = 4.24$ (start value, 4 M&Ms)

$b = 1.49$ (150%) ← Adding about $\frac{1}{2}$ of existing M&Ms to the cup.

(b) It is unlikely your data is "perfect." If this experiment were to work out exactly like theory, what would be the values of a and b?

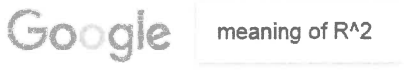
$a = 4$

$b = 1.5$

(c) Click on residuals (Desmos should plot them). What are residuals?

observed value minus the fitted value

(d) You'll also notice an R^2 value is provided. Do a quick search on its meaning—for example,



Interpret the R^2 that you obtained from the M&M data. Does the model fit the data well?

$R^2 \approx 0.9984$

(how closely the data fit the regression curve; what percent of the variation in y is explained by x)

8. Make a projection (this is a big part of data collection and statistics). Suppose you had a large number of M&Ms and you could conduct many trials. How many M&Ms would you expect on the 10th trial? Don't carry this out; use reasoning to make a prediction.

$y = 4(1.5)^x$ let $x = 10$

$y = 4(1.5)^{10} \approx 231$ (about 231 M&Ms)

Part II: Watch the Function Decrease

Repeat Part I but start with 40 M&Ms. (You can start a new session in Desmos or just advance to the next command line.) With each trial, *remove* those M&Ms showing an M, put the others back into the cup, and repeat. Record this data in a new table and analyze this regression model.

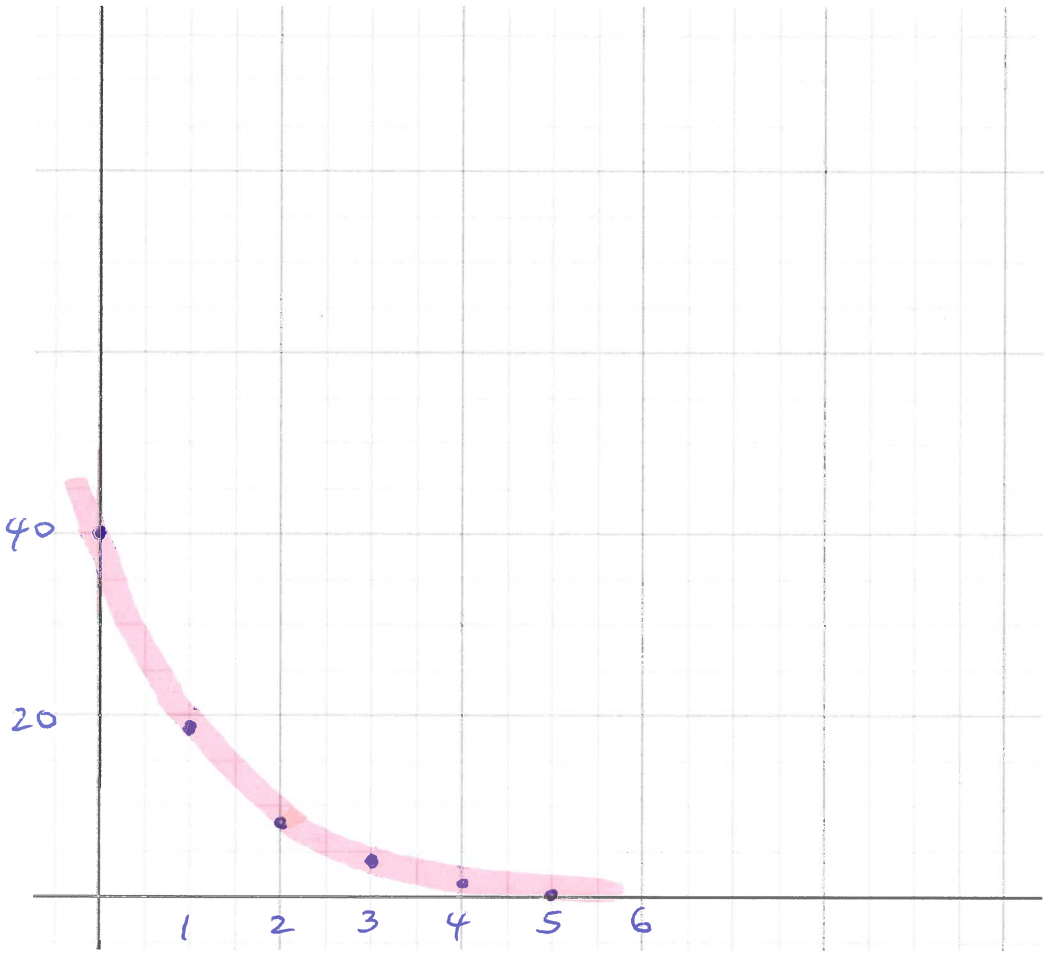
1. Explain the meaning of the new values a and b in the model $y_2 \sim ab^{x_2}$.

$y \approx 40.13(.46)^x$

$a \approx 40$ (start value, 40 M&Ms)

$b \approx 50\% = 0.5$ (removing this many M&Ms per trial)

2. Show a plot of the data and regression curve. Again, pick an appropriate scale on the x and y axes (see next page).



x_2	y_2
0	40
1	19
2	8
3	4
4	2
5	0

3. What are the "true" theoretical values of a and b?

$a = 40$ (start w/ 40 M&Ms)

$b = \frac{1}{2} = 0.5$

4. What is the value of R^2 here? How well does the model fit the data?

$R^2 = 0.99 \approx 1$ (very good fit)

5. At approximately what point would you expect to run out of M&Ms? How do you know?

x	0	1	2	3	4	5	6
y	40	20	10	5	3	1	0

About trial # 6