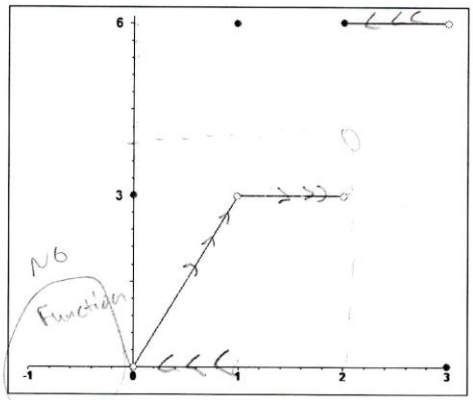
MTH 150
Exam 1
Fall 2014

DIRECTIONS: This is a closed book, closed notes exam. No electronic devices are allowed (this means calculators, computers, cell phones, pagers, etc.). Be neat and show all work to receive full credit. Correct answers without the supporting evidence to back it up receive only partial credit. Good luck.

For problems 1-12, consider the function $y = g(x)$ graphed below. Notice that the domain of this function is $[0,3]$. Fill in the blanks below based on the diagram or write DNE for "does not exist." Each problem is worth **2 points**.



1. $\lim_{x\to 1^+} g(x) =$ 3
2. $\lim_{x\to 1^-} g(x) =$ 3
3. $\lim_{x\to 1} g(x) =$ 3
4. $g(1) =$ 6
5. $\lim_{x\to 2^+} g(x) =$ 6
6. $\lim_{x\to 2^-} g(x) =$ 3
7. $\lim_{x\to 2} g(x) =$ DNE
8. $g(2) =$ 6
9. $\lim_{x\to 0} g(x) =$ DNE No left-side limit
10. $g(0) =$ 3
11. True/False: The function $y = g(x)$ is continuous on $[0,3]$. F
12. True/False: There exists a c on $[0,3]$ where $g(c) = 4$. F

-0

For exercises 13-16, evaluate the limit analytically. In other words, use some algebra or a well-known Calculus result to arrive at your answer.

13. **(8 points)** $\lim_{x\to 1}\left(3x^2 - e^x\right)$

$\lim_{x\to 1} 3(1)^2 - e^1$

$\boxed{3 - e}$ ✓ $\quad 3 - 2.17 \approx \boxed{.83}$

14. **(8 points)** $\lim_{h\to -3}\dfrac{h^2+2h-3}{h^2+4h+3}$ $= \lim_{h\to -3}\dfrac{(h+3)(h-1)}{(h+3)(h+1)}$ ✓ $\quad h+3=0$ $\quad h=-3$ hole

$\lim_{h\to -3}\dfrac{((-3)-1)}{((-3)+1)}$

$= \dfrac{-4}{-2}$

$= \boxed{2}$ ✓

15. **(8 points)** $\lim_{t\to 0}\dfrac{t}{\sqrt{2t+1}-1}$ $\cdot \dfrac{\sqrt{2t+1}+1}{\sqrt{2t+1}+1}$

$= \lim_{t\to 0}\dfrac{t\left(\sqrt{2t+1}+1\right)}{2t+1-1}$ ✓ $= \lim_{t\to 0}\dfrac{\sqrt{2t+1}+1}{2}$

$= \lim_{t\to 0}\dfrac{t\left(\sqrt{2t+1}+1\right)}{2t}$ ✓ $= \dfrac{\sqrt{2(0)+1}+1}{2}$

$= \dfrac{2}{2} = \boxed{1}$ ✓

16. **(8 points)** $\lim_{\theta\to \pi/4}\dfrac{\sin\theta - \cos\theta}{1-\tan\theta}$

$\frac{0}{0}$ form

$\lim_{\theta\to\pi/4}\dfrac{\sin\theta - \cos\theta}{1-\frac{\sin\theta}{\cos\theta}} \cdot \cos\theta$ $= \lim_{\theta\to\pi/4}\dfrac{(\sin\theta - \cos\theta)\cos\theta}{\cos\theta - \sin\theta}$ $\quad -1$

$= \lim_{\theta\to\pi/4}(-\cos\theta)$

$= -\cos\pi/4 = \boxed{-\sqrt{2}/2}$

Problems 17-19 are multiple choice; no work is necessary. Circle the appropriate letter.

17. **(3 points)** The function $f(x)=x^3+x-1$ is continuous on the interval $[1,2]$. Which of the following is true by the Intermediate Value Theorem for some c in $[1,2]$?

A. $f(c)=0$ C. $f(c)=5$ E. none of these

B. $f(c)=-2$ D. $f(c)=13$

18. **(3 points)** Which of the following viewing screens (if any) suggest that $\lim_{x\to 4} f(x)=7$?

A.

X	Y1
3.97	7
3.98	7
3.99	7
[redacted]	7
4.01	7
4.02	7
4.03	7

X=4

B.

X	Y1
6.97	4.09
6.98	4.06
6.99	4.03
[redacted]	4
7.01	3.97
7.02	3.94
7.03	3.91

X=7

C.

X	Y1
3.97	4.0009
3.98	4.0004
3.99	4.0001
[redacted]	7
4.01	4.0001
4.02	4.0004
4.03	4.0009

X=4

D.

X	Y1
3.97	6.94
3.98	6.96
3.99	6.98
[redacted]	ERROR
4.01	7.02
4.02	7.04
4.03	7.06

X=4

E. none of these

19. **(3 points)** Find the x–value(s) at which $f(x)=\dfrac{x}{x^2-2x}$ is not continuous and choose the answer that best classifies the nature of the discontinuity(ies).

~~A. only $x=2$ (limit does not exist)~~

~~B. $x=0$ (limit does not exist); $x=2$ (limit does not exist)~~

C. $x=0$ (limit does exist); $x=2$ (limit does not exist)

D. $x=0$ (limit does not exist); $x=2$ (limit does exist)

~~E. only $x=0$ (limit does exist)~~

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DIRECTIONS: Calculators are permitted on this part of the exam. However, answers based solely on calculator results are unacceptable. You must still <u>show all work</u> to receive full credit. Good luck.

20. **(5 points)** Use a table the determine $\lim_{x\to 0}\frac{e^{2x}-2x-1}{x^2}$. Explain how you arrived at your conclusion.

$\lim_{x\to 0}\frac{e^{2x}-2x-1}{x^2} = 2$ ✓

As the value of x approaches 0, the value of y approaches 2.

x	y
-.003	1.994
-.002	1.996
-.001	1.997
0	
.001	2.001
.002	2.002
.003	2.004

✓

21. **(8 points)** Consider the function $h(x)=\frac{|x-7|}{x-7}$. Does $\lim_{x\to 7}h(x)$ exist? If so, find the limit. If not, explain why not.

$\lim_{x\to 7}h(x)$ does not exist because there is a jump in the graph at 7. As you approach 7 from the right, the limit is 1, but as you approach 7 from the left, the limit is -1. Because you approach 2 different values, the limit at x=7 fails to exist. ✓

✓

Good!

-0

22. Consider the function $f(x)=\dfrac{x^2+2x-15}{x^2-9}$.

$\dfrac{(x-3)(x+5)}{(x-3)(x+3)}$ hole @ $x=3$ V.A @ $x=-3$

(a) **(5 points)** Find the value(s) of x at which $f(x)$ is discontinuous and label each discontinuity as removable (having a limit) or non-removable (no limit exists).

hole @ $x=3$ (removable) ✓ $\lim_{x\to 3}\frac{(x+5)}{(x+3)}=\frac{3+5}{3+3}=\frac{8}{6}=\frac{4}{3}$

vertical asymptote @ $x=-3$ (non-removable) ✓ $\lim_{x\to -3}\frac{(x+5)}{(x+3)}$ = DNE

x	y
-3.002	-999
-3.001	-1999
-3	error
-2.999	2001
-2.998	1001

$\lim_{x\to -3^-} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$

(b) **(4 points)** Write down an appropriate limit statement to show that $f(x)$ has a vertical asymptote. Give an equation for the VA.

Good

Limit: $\lim_{x\to -3} f(x)$ = DNE ✓ **VA:** $x=-3$ ✓

(c) **(4 points)** Write down an appropriate limit statement to show that $f(x)$ has a horizontal asymptote. Give an equation for the HA.

x	y
0	1.6667
1000	1.002
10,000	1.0002
20,000	1.0001

Limit: $\lim_{x\to\infty} f(x) = 1$ ✓ **HA:** $y=1$ ✓

$\lim_{x\to\infty}\left(\frac{x+5}{x+3}\right)$

$\frac{x}{x}=\frac{1}{1}=1$

- limit properties

23. **(9 points)** Prove that $\lim_{x\to\infty}\dfrac{\sin x+\cos x}{x}=0$. Provide all supporting work.

Note that $-1\le \sin x\le 1$ & $-1\le \cos x\le 1$ for all x. Thus,

$$-2\le \sin x+\cos x\le 2$$

since $x>0$ $(x\to\infty)$,

$$-\frac{2}{x}\le\frac{\sin x+\cos x}{x}\le\frac{2}{x}$$

Now let $x\to\infty$ so $\frac{2}{x}\to 0$, $-\frac{2}{x}\to 0$.

Thus, $\lim_{x\to\infty}\frac{\sin x+\cos x}{x}=0$ (Squeeze Thm)