MTH 150
Exam 3
Fall 2014

Name: KEY

DIRECTIONS: This is a closed book, closed notes exam. No electronic devices are allowed (this means calculators, computers, cell phones, pagers, etc.). You do not have to show any work on this part of the test. Good luck.

1. **(20 points)** Match each statement, theorem, definition, or term with the letter of the statement that describes it. Letters are used only once.

C Extreme Value Theorem

D Rolle's Theorem

J Mean Value Theorem

A f is concave upward.

E f is increasing.

F f is concave downward.

I f is decreasing.

H relative extremum

B point of inflection

G critical number

A. f' is increasing.

~~B~~. f'' switches sign.

~~C~~. If a function is continuous on a closed interval, then it has a maximum and minimum on the interval.

~~D~~. Guarantees a point of horizontal tangency.

~~E~~. f' is positive.

F. f' is decreasing.

~~G~~. x-value for which f' is undefined or equal to zero.

~~H~~. f' switches sign.

~~I~~. f' is negative.

~~J~~. Guarantees a point where the slope is the average rate of change.

Problems 2-5: Circle the letter that corresponds to your answer. **(3 points each)**

2. Find the value of the absolute minimum of $f(x) = x^2 + 6x$ on the interval $[-4, -1]$.

A. –4
B. –1
C. –8
D. –5
E. 3

$f'(x) = 2x + 6 = 0$

$x = -3$

x	$f(x)$
-3	-9
-4	-8
-1	-5

Whoops! Answer should be -9

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3. Choose the **best** answer….The Second Derivative Test is used to…

A. determine intervals of increase/decrease
B. determine the concavity of the graph
C. find relative extrema
D. find points of inflection
E. None of these

C

$h'(x) = \frac{(x-3)1 - x\cdot 1}{(x-3)^2} = \frac{-3}{(x-3)^2}$

$h''(x) = -3(-2)(x-3)^{-3} = \frac{6}{(x-3)^3}$

4. Consider the function $h(x) = \frac{x}{x-3}$. Find the interval(s) on which $h(x)$ is concave upward.

A. $(-\infty,3)\cup(3,\infty)$
B. $(-\infty,3)$ only
C. $(3,\infty)$ only
D. $(-3,3)$ only
E. The graph is never concave upward.

C

$-$ $+$ h''

3 CC↑

5. Imagine that you increase the dimensions of a square with side x_1 to a square with side length x_2. The change in the area of the square ΔA is approximated by the differential dA. In this example, dA is

A. $(x_2 - x_1)2x_1$
B. $(x_2 - x_1)2x_2$
C. $x_2^2 - x_1^2$
D. $(x_2 - x_1)^2$
E. None of these

A

$A = x^2$

$\frac{dA}{dx} = 2x$

$dA = 2x\ dx$

$= 2x_1(x_2 - x_1)$

6. Consider the function $f(x) = \sin(4x)$.

(a) **(5 points)** Explain how to find an antiderivative of $f(x)$.

Find a function w/ derivative $\sin 4x$

$\left(-\frac{1}{4}\cos 4x\right)' = -\frac{1}{4}(-\sin 4x)\cdot 4$

$= \sin 4x$

ANS: $-\frac{1}{4}\cos 4x$

(b) **(3 points)** Express the above result using integral $\int$ notation, $f(x)$, and the antiderivative into one concise, valid mathematical statement.

$\int \sin 4x\ dx = -\frac{1}{4}\cos 4x + C$

7. **(8 points)** A plane begins its takeoff at 2:00 P.M. on a 2500 mile flight. After 5.5 hours, the plane arrives at its destination. Explain why there are at least two times during the flight when the speed of the plane is 400 mph. Note that $\frac{2500}{5.5} \approx 455$.

455 ≈ AROC over entire trip.

MVT guarantees at least one instant where IROC = AROC. Thus, there is at least one point when the plane was traveling at precisely 455 mph.

Thus, it must have passed through 400 mph twice — once on its ascent, once again on its descent.

8. **(8 points)** Determine the value of $\lim_{x\to 0} \frac{e^x - \frac{1}{2}x^2 - x - 1}{x^3}$, if possible. $\frac{0}{0}$

$$\lim_{x\to 0} \frac{e^x - x - 1}{3x^2} \qquad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{e^x - 1}{6x} \qquad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{e^x}{6}$$

$$= \frac{1}{6}$$

BONUS (5 points):

L'Hopital's Rule should really be credited to a *different* individual—not L'Hopital. Who?

Bernoulli

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DIRECTIONS: Calculators are permitted on this part of the exam. However, answers based solely on calculator results are unacceptable. You must still <u>show all work</u> to receive full credit. Good luck.

$f' = 3x^2 - 6x$

9. Consider the function $f(x) = x^3 - 3x^2 + 3$. $f'' = 6x - 6$

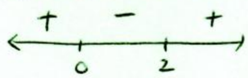
(a) **(2 points)** Find all of the critical numbers of f.

$3x(x-2) = 0$

$x = 0, 2$

0, 2

(b) **(3 points)** Find the intervals on which f is increasing and decreasing.



INC: $(-\infty, 0)$; $(2, \infty)$
DEC: $(0, 2)$

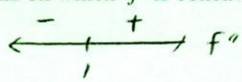
(c) **(2 points)** Locate any relative extrema.

max @ $(0, 3)$
min @ $(2, -1)$

(d) **(3 points)** Find the intervals on which f is concave upward and concave downward.

$6x - 6 = 0$

$x = 1$



CC ↑ : $(1, \infty)$
CC ↓ : $(-\infty, 1)$

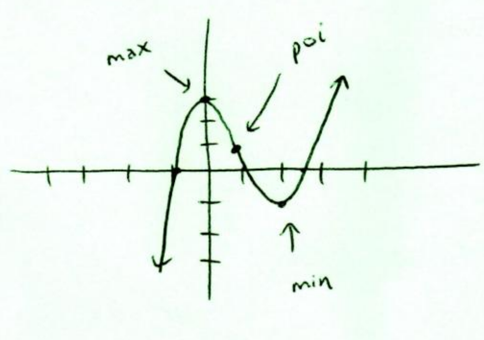
(e) **(3 points)** Find any points of inflection.

$(1, 1)$

(f) **(3 points)** Approximate any x-intercept(s). Use the calculator.

$-.879$, 1.347, 2.532

(g) **(4 points)** Make a complete, clear, neat sketch of the graph of f by using parts (b)-(f) from above.



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10 . **(8 points)** Find the critical numbers of the function $f(x) = xe^{-x}$, if any. Additionally, find the open intervals on which the function is increasing or decreasing and locate all relative extrema.

$f(x) = xe^{-x}$

$f'(x) = xe^{-x}(-1) + e^{-x}$

$= e^{-x}(1-x)$

$\boxed{x = 1}$

f': $+$ on the left of 1, $-$ on the right of 1

INC DEC

INC: $(-\infty, 1)$

DEC: $(1, \infty)$

max @ $(1, e^{-1})$

11. **(8 points)** Use Newton's method $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ to approximate the zero of the function $f(x) = x^3 - 3x + 4$. Use $x_0 = -2$ as an initial guess and find x_1, x_2, and x_3. Show the work/set-up for the computation of x_1; otherwise let the calculator do the work.

$f(x) = x^3 - 3x + 4$

$f'(x) = 3x^2 - 3$

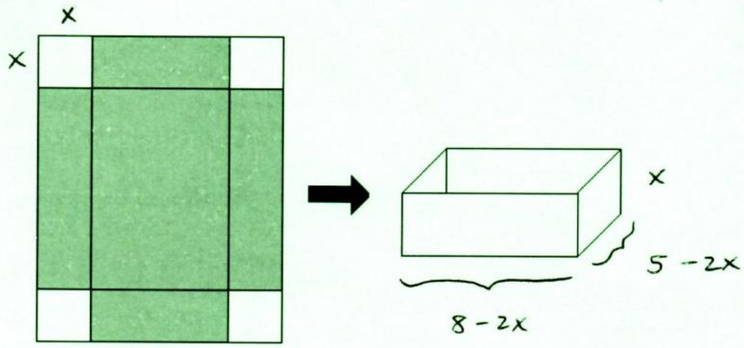
$$x_n - \frac{x_n^3 - 3x_n + 4}{3x_n^2 - 3}$$

$x_1 = -2.222$

$x_2 = -2.196$

$x_3 = -2.196$

12. **(8 points)** A piece of sheet metal is rectangular, 5 ft wide and 8 ft long. Congruent squares (same size) are to be cut from its four corners. The resulting piece of metal is to be folded and welded to form a box with an open top, as shown below. How should this be done to get a box of largest possible volume? **Be sure to show all work here.**



$$\text{Vol} = x(5-2x)(8-2x), \quad x > 0, \quad x < 2.5$$

$$\begin{aligned} V(x) &= (5x - 2x^2)(8 - 2x) \\ &= 40x - 10x^2 - 16x^2 + 4x^3 \\ &= 40x - 26x^2 + 4x^3 \end{aligned}$$

$$V'(x) = 40 - 52x + 12x^2 \overset{\text{set}}{=} 0$$

$$4(10 - 13x + 3x^2) = 0$$

$$4(3x^2 - 13x + 10) = 0$$

$$4(3x - 10)(x - 1) = 0$$

$\cancel{x = 10/3}$ Not in domain $\qquad x = 1$

V': $+$ on $(0, 1)$, $-$ on $(1, 2.5)$

0 1 2.5

max @ $x = 1$

(max volume)

Sides are

$6 \times 3 \times 1$

$= 18 \text{ ft}^3$