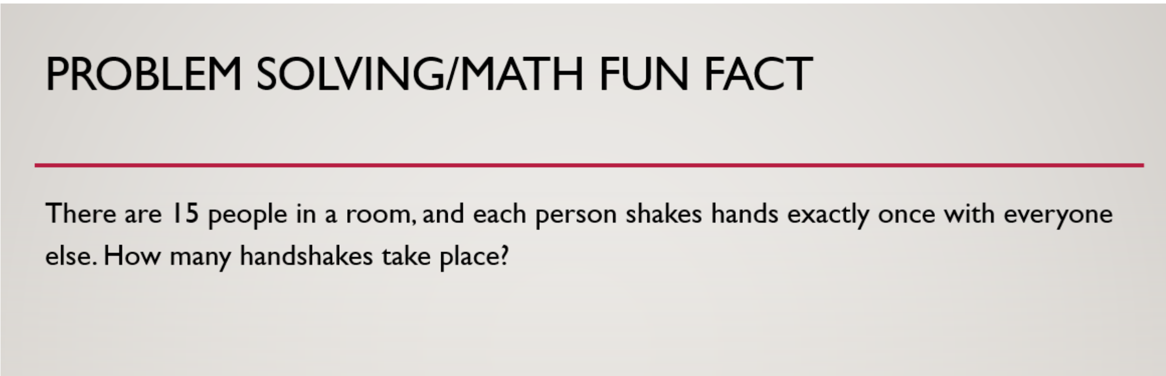
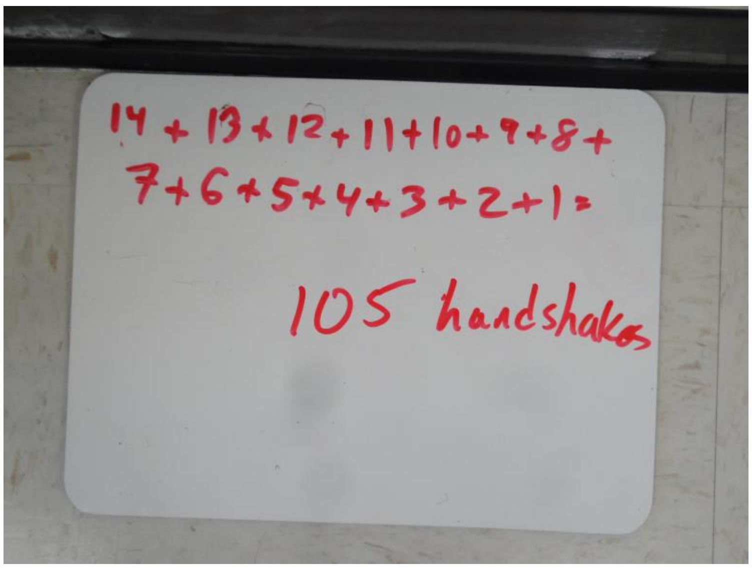
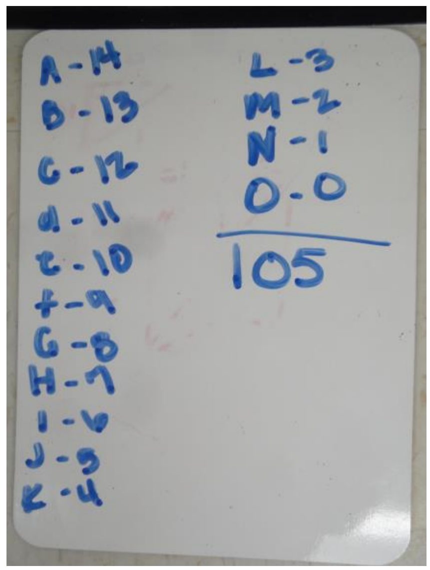
May 22, 2017

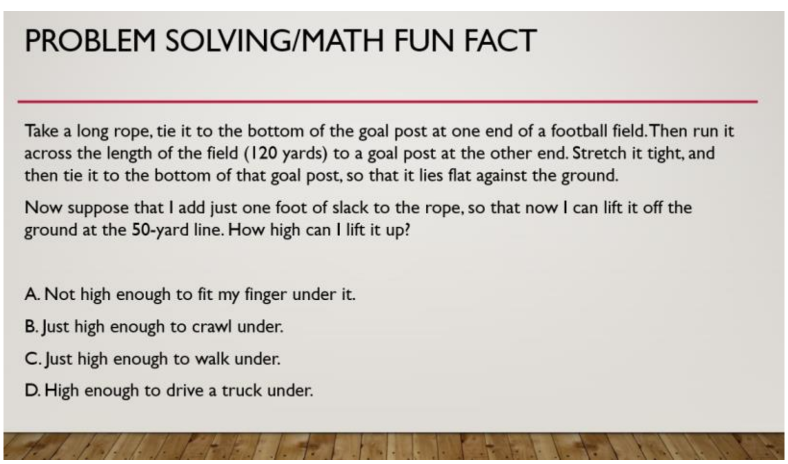
Warm Up Activities:



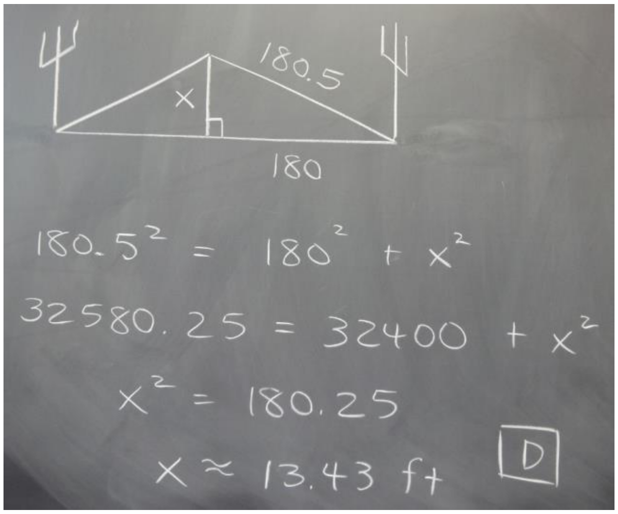




We talked briefly about a snazzy way to find the number of handshakes: 14 + 1 = 15, 13 + 2 = 15, 12 + 3 = 15, etc. You will find seven pairs of 15 or $7 \cdot 15 = 105$ handshakes.



I forgot to take a picture in class so here is my work (the same as what we saw in class):



1.4: The Tangent and Velocity Problems

Time interval	[2,2.5]	[2.25,2.5]	[2.4,2.5]	[2.49,2.5]	[2.4999,2.5]
Average velocity					

Consider dropping a ball from the top of a 100 foot parking deck. The height $h(t)$ of the ball t seconds after it is dropped is modeled by $h(t) = 100 - 16t^2$. It can be shown that the ball hits the ground after $t = 2.5$ seconds (How?). The average speed of the ball over any time period is given by $r = \frac{\Delta h}{\Delta t} = \frac{\text{change in height}}{\text{change in time}}$. If we wished to calculate, on average, how fast the ball was moving in its last half-second, this would be

$$r = \frac{\Delta h}{\Delta t} = \frac{\text{change in height}}{\text{change in time}} = \frac{h(2.5) - h(2)}{2.5 - 2} = -72 \text{ feet/sec.}$$

Complete the table above and answer the question:

How fast is the ball traveling at the moment of impact?

example 1:

time	(2, 2.5)	(2.25, 2.5)	(2.4, 2.5)	(2.49, 2.5)	(2.4999, 2.5)
A. velo.	-72	-76	-78.4	-79.84	-79.9984

$h(t) = 100 - 16t^2$

1. $r = \frac{\Delta h}{\Delta t} = \frac{h(2.5) - h(2)}{2.5 - 2} = -72$

2. $r = \frac{\Delta h}{\Delta t} = \frac{100 - 16(2.5)^2 - [100 - 16(2.25)^2}{2.5 - 2.25}$

Important: Each calculation in the table represents an **average** velocity (we are using two distinct points to calculate the velocity). Only when we arrive at the interval $[2.4999, 2.5]$ can we really interpret this as an **instantaneous** velocity (because the two points are so close to one another). Geometrically, when we calculate an average through two points, we are calculating the slope of the secant line passing through these points. When the two points become one, the secant line becomes a tangent line *whose slope is an instantaneous rate of change* (in this case, the *velocity* at a precise moment in time).

1.5: The Limit of a Function

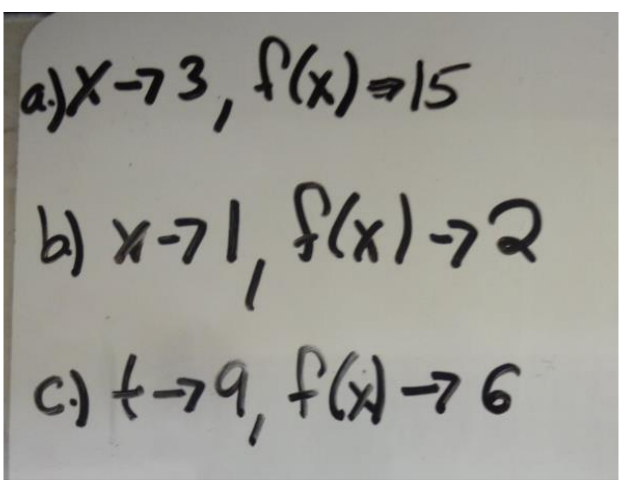
PROBLEM

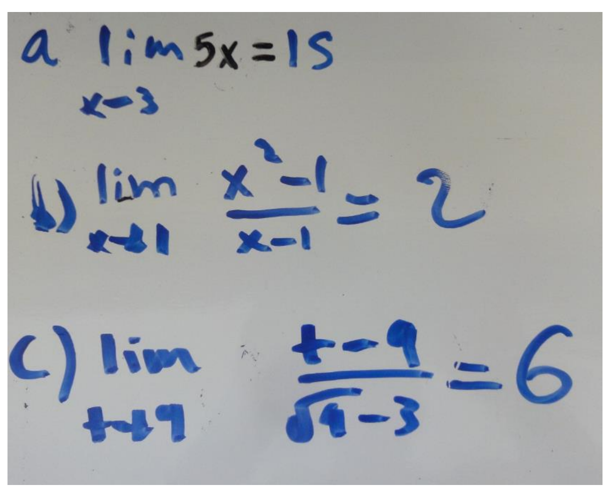
Use a table (e.g., your calculator) to determine, if possible, the following:

(a) $\lim_{x\to 3} 5x$

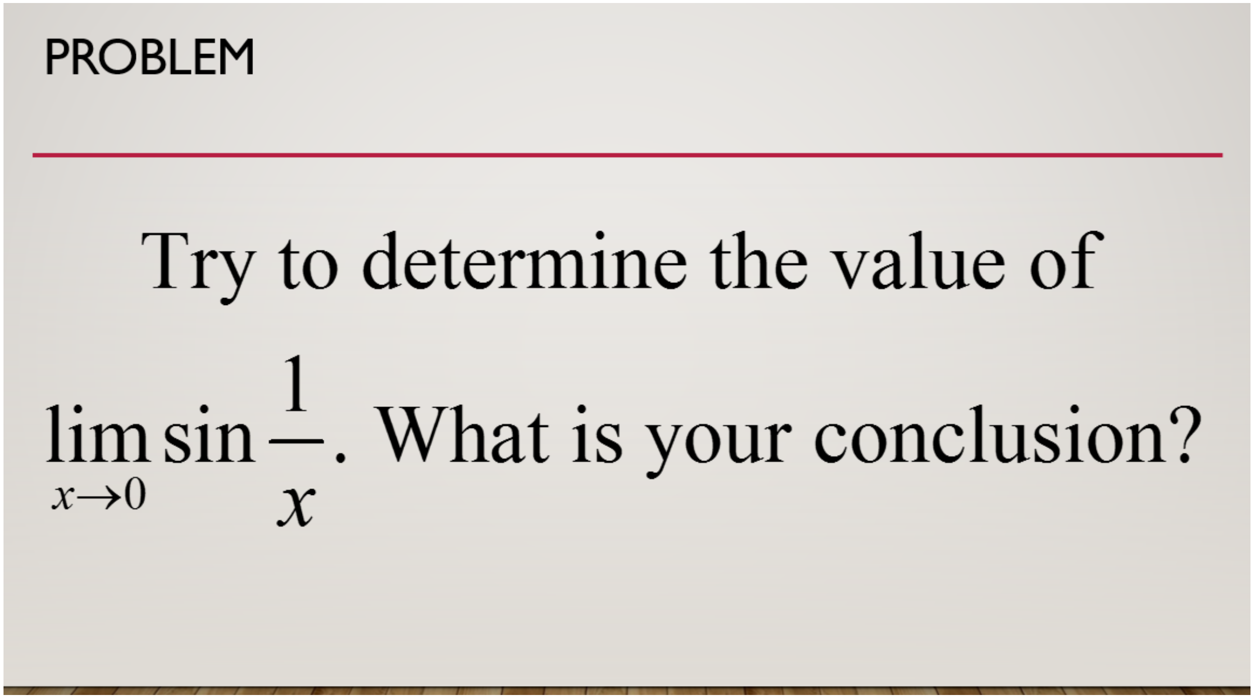
(b) $\lim_{x\to 1}\frac{x^2-1}{x-1}$

(c) $\lim_{t\to 9}\frac{t-9}{\sqrt{t}-3}$



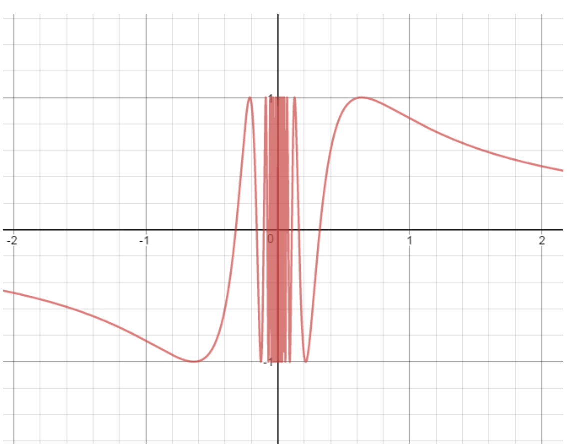


The above work is obtained by using the TABLE feature on the calculator. A clustering of values in the Y column assures the existence of a limit.

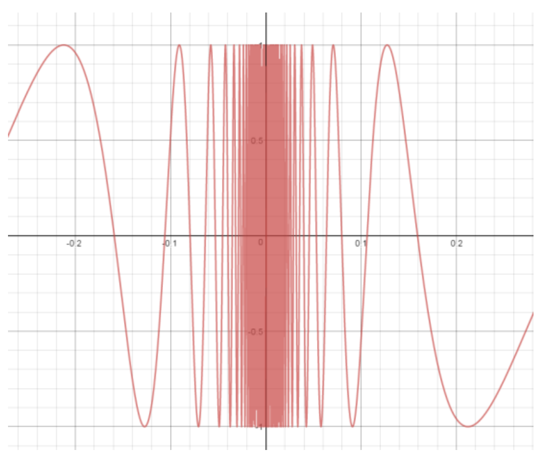


A nice way to approach this is to examine the graph of $y = \sin\left(\frac{1}{x}\right)$ near $x = 0$. The closer you get to zero, the harder it is to see what is going on with $\sin\left(\frac{1}{x}\right)$:

A close look:

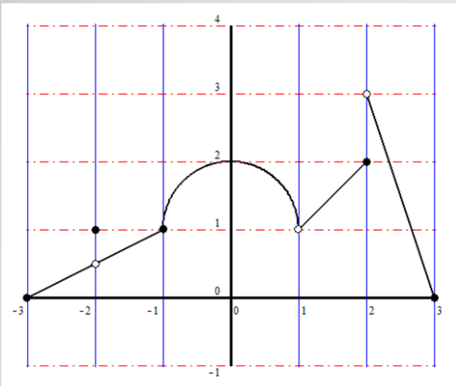


An even closer look:



Due to the increased oscillation as one gets closer and closer to $x = 0$, the limit fails to exist.

FILL IN THE BLANKS AS APPROPRIATE. BE PREPARED TO EXPLAIN YOUR ANSWERS!!



$\lim\limits_{x\to -2^-} f(x) =$ ____ $\qquad \lim\limits_{x\to 2^-} f(x) =$ ____

$\lim\limits_{x\to -2^+} f(x) =$ ____ $\qquad \lim\limits_{x\to 2^+} f(x) =$ ____

$\lim\limits_{x\to -2} f(x) =$ ____ $\qquad \lim\limits_{x\to 2} f(x) =$ ____

$f(-2) =$ ____ $\qquad f(2) =$ ____

$x \to -2^-$, $f(x) \to 1/2$ $\qquad x \to 2^-$, $f(x) = 2$

$x \to -2^+$, $f(x) \to 1/2$ $\qquad x \to 2^+$, $f(x) \to 3$

$x \to -2$, $f(x) \to 1/2$ $\qquad x \to 2$, $f(x) =$ DNE

$f(-2) = 1$ $\qquad f(2) = 2$

Just a quick note here: For the solutions on the right, either write something like

As $x \to 2^-$, $f(x) \to 2$ (using arrow notation)

or

$\lim\limits_{x\to 2^-} f(x) = 2$.