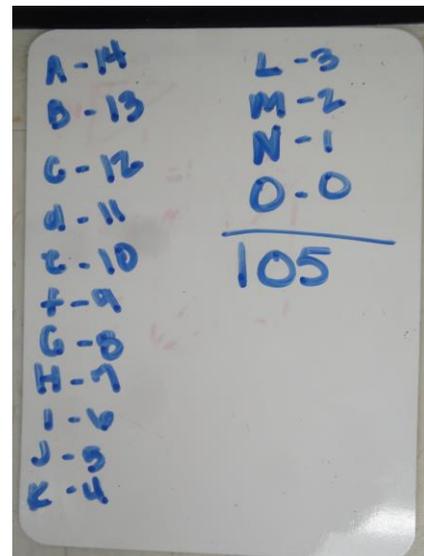
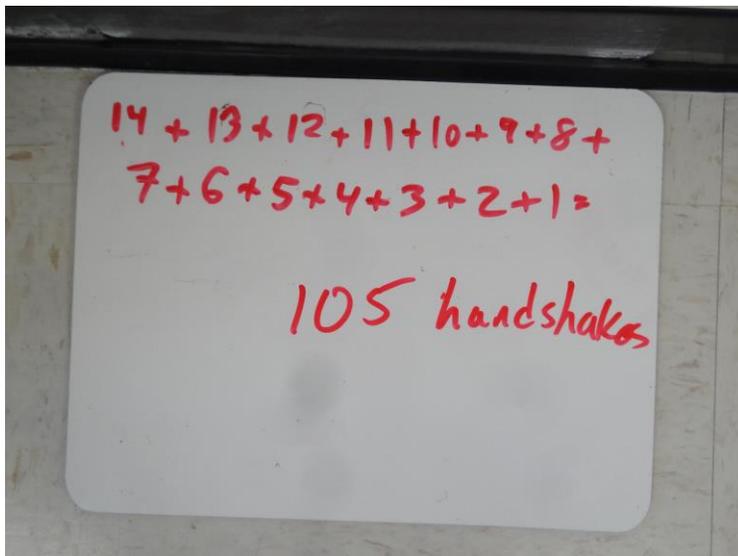


May 22, 2017

Warm Up Activities:

PROBLEM SOLVING/MATH FUN FACT

There are 15 people in a room, and each person shakes hands exactly once with everyone else. How many handshakes take place?



We talked briefly about a snazzy way to find the number of handshakes: $14 + 1 = 15$, $13 + 2 = 15$, $12 + 3 = 15$, etc. You will find seven pairs of 15 or $7 \cdot 15 = 105$ handshakes.

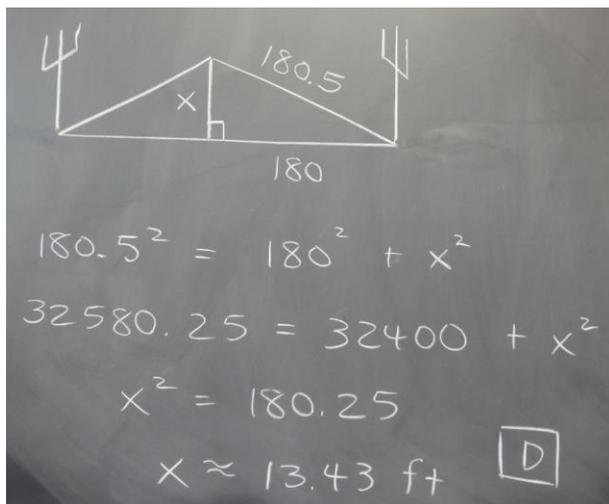
PROBLEM SOLVING/MATH FUN FACT

Take a long rope, tie it to the bottom of the goal post at one end of a football field. Then run it across the length of the field (120 yards) to a goal post at the other end. Stretch it tight, and then tie it to the bottom of that goal post, so that it lies flat against the ground.

Now suppose that I add just one foot of slack to the rope, so that now I can lift it off the ground at the 50-yard line. How high can I lift it up?

- A. Not high enough to fit my finger under it.
- B. Just high enough to crawl under.
- C. Just high enough to walk under.
- D. High enough to drive a truck under.

I forgot to take a picture in class so here is my work (the same as what we saw in class):



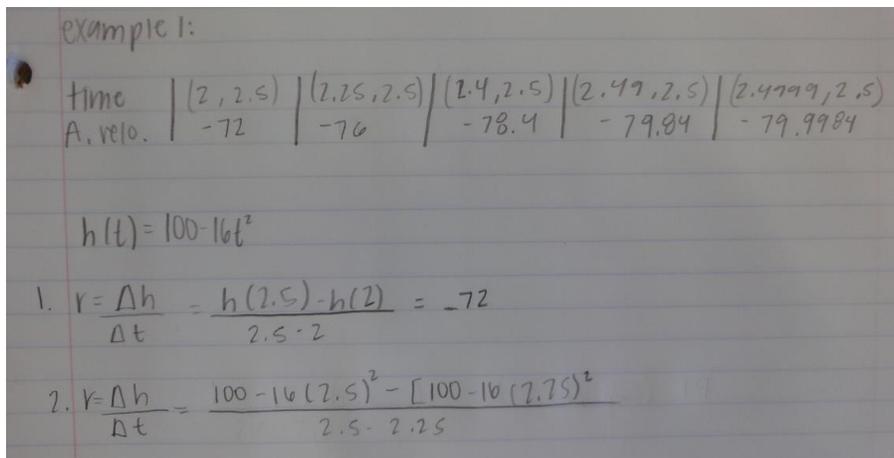
1.4: The Tangent and Velocity Problems

Time interval	[2,2.5]	[2.25,2.5]	[2.4,2.5]	[2.49,2.5]	[2.4999,2.5]
Average velocity					

Consider dropping a ball from the top of a 100 foot parking deck. The height $h(t)$ of the ball t seconds after it is dropped is modeled by $h(t) = 100 - 16t^2$. It can be shown that the ball hits the ground after $t = 2.5$ seconds (How?). The average speed of the ball over any time period is given by $r = \frac{\Delta h}{\Delta t} = \frac{\text{change in height}}{\text{change in time}}$. If we wished to calculate, on average, how fast the ball was moving in its last half-second, this would be

$$r = \frac{\Delta h}{\Delta t} = \frac{\text{change in height}}{\text{change in time}} = \frac{h(2.5) - h(2)}{2.5 - 2} = -72 \text{ feet/sec.}$$

Complete the table above and answer the question:
How fast is the ball traveling at the moment of impact?



Important: Each calculation in the table represents an **average** velocity (we are using two distinct points to calculate the velocity). Only when we arrive at the interval $[2.4999, 2.5]$ can we really interpret this as an **instantaneous** velocity (because the two points are so close to one another). Geometrically, when we calculate an average through two points, we are calculating the slope of the secant line passing through these points. When the two points become one, the secant line becomes a tangent line *whose slope is an instantaneous rate of change* (in this case, the *velocity* at a precise moment in time).

1.5: The Limit of a Function

PROBLEM

Use a table (e.g., your calculator) to determine, if possible, the following:

(a) $\lim_{x \rightarrow 3} 5x$

(b) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

(c) $\lim_{t \rightarrow 9} \frac{t - 9}{\sqrt{t} - 3}$

Handwritten work for problem 1.5 showing the results of a calculator table for each limit:

- a) $x \rightarrow 3, f(x) \rightarrow 15$
- b) $x \rightarrow 1, f(x) \rightarrow 2$
- c) $t \rightarrow 9, f(x) \rightarrow 6$

Handwritten work for problem 1.5 showing the algebraic solutions for each limit:

- a) $\lim_{x \rightarrow 3} 5x = 15$
- b) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2$
- c) $\lim_{t \rightarrow 9} \frac{t - 9}{\sqrt{t} - 3} = 6$

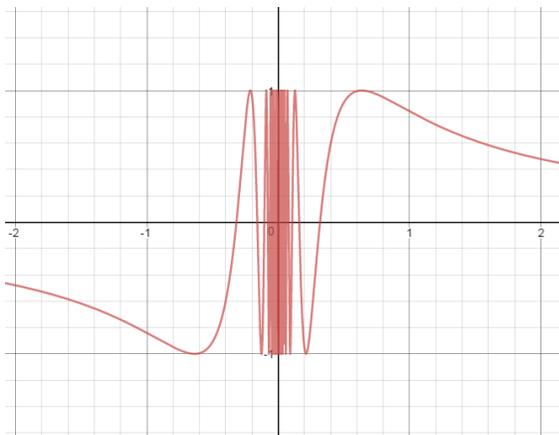
The above work is obtained by using the TABLE feature on the calculator. A clustering of values in the Y column assures the existence of a limit.

PROBLEM

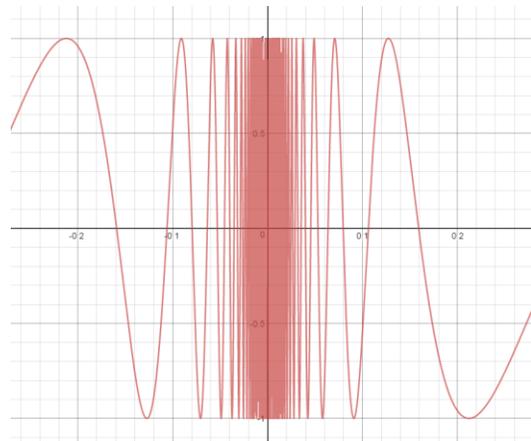
Try to determine the value of $\lim_{x \rightarrow 0} \sin \frac{1}{x}$. What is your conclusion?

A nice way to approach this is to examine the graph of $y = \sin\left(\frac{1}{x}\right)$ near $x = 0$. The closer you get to zero, the harder it is to see what is going on with $\sin\left(\frac{1}{x}\right)$:

A close look:

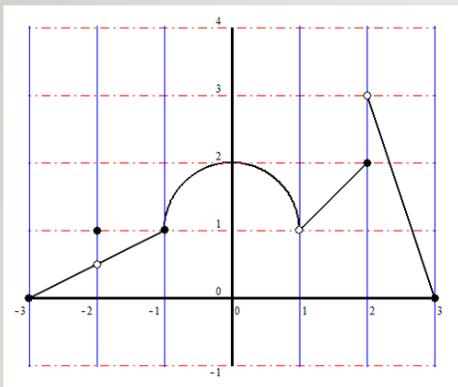


An even closer look:



Due to the increased oscillation as one gets closer and closer to $x = 0$, the limit fails to exist.

FILL IN THE BLANKS AS APPROPRIATE. BE PREPARED TO EXPLAIN YOUR ANSWERS!!



$$\lim_{x \rightarrow -2^-} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 2^-} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow -2^+} f(x) = \underline{\hspace{2cm}}$$

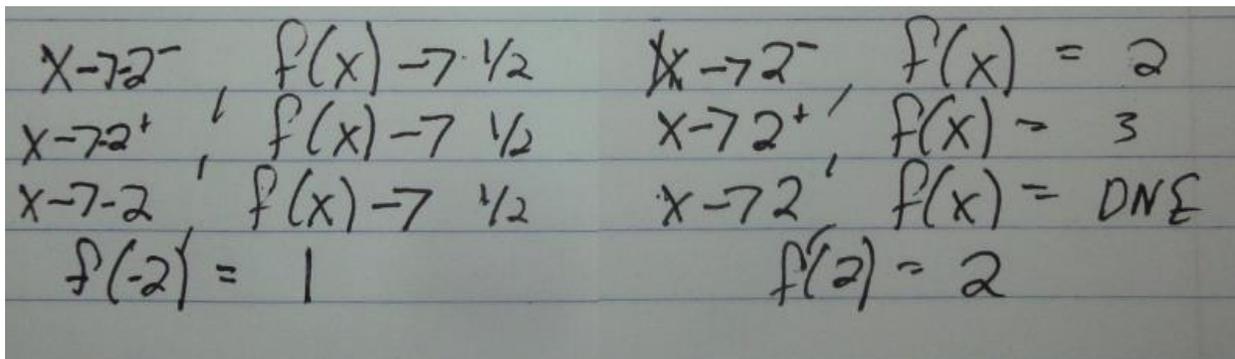
$$\lim_{x \rightarrow 2^+} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow -2} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 2} f(x) = \underline{\hspace{2cm}}$$

$$f(-2) = \underline{\hspace{2cm}}$$

$$f(2) = \underline{\hspace{2cm}}$$



Just a quick note here: For the solutions on the right, either write something like

As $x \rightarrow 2^-$, $f(x) \rightarrow 2$ (using arrow notation)

or

$$\lim_{x \rightarrow 2^-} f(x) = 2.$$