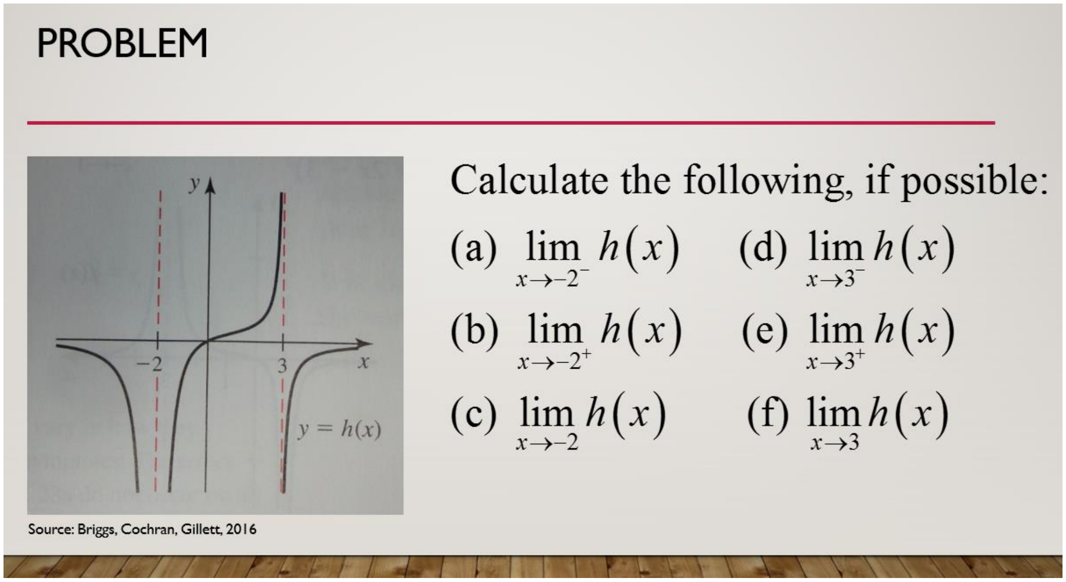
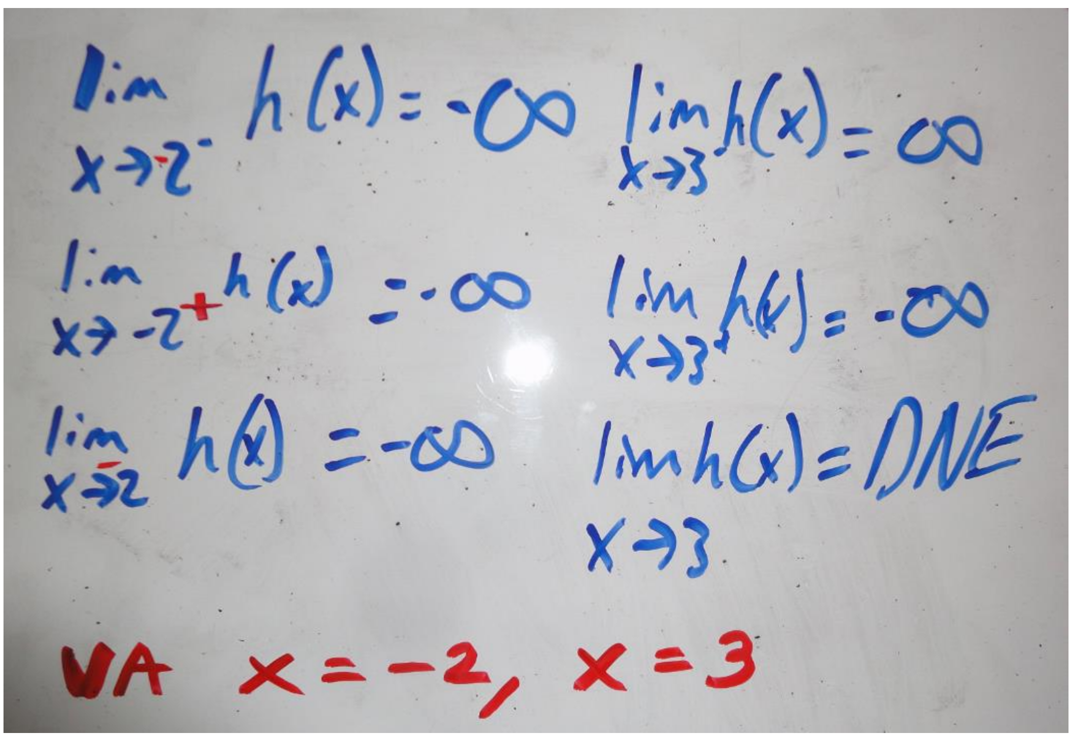
May 23, 2017

1.5B: Infinite Limits/Vertical Asymptotes

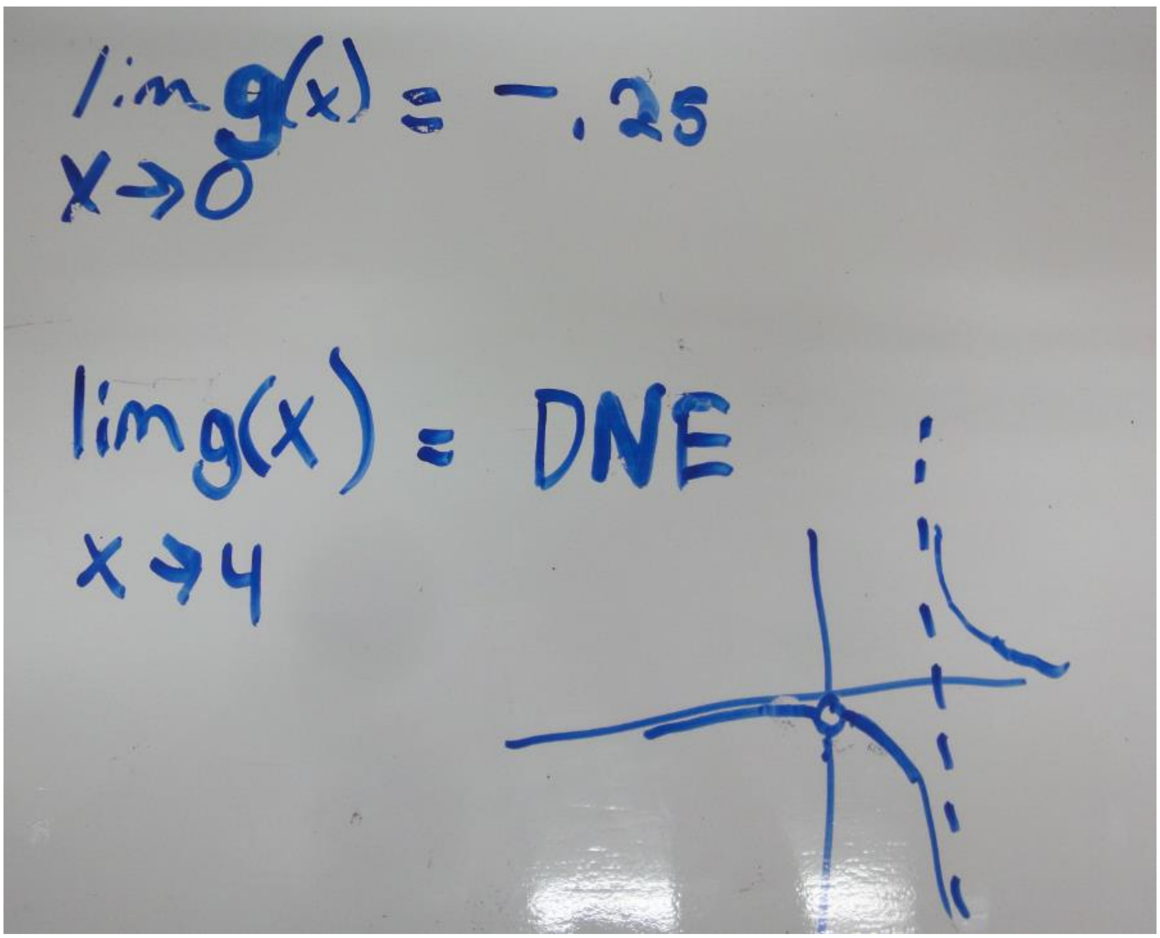




All of the above could be answered with a "does not exist" but using ∞ and $-\infty$ is preferred since it is more descriptive.

PROBLEM

Consider the function $g(x)=\dfrac{x}{x^2-4x}$. Examine $\lim_{x\to 0} g(x)$ and $\lim_{x\to 4} g(x)$ and describe what is happening in each case.



Near $x=0$: The function tends to -0.25 on either side of zero (notice the hole/discontinuity in the graph at $(0,-1/4)$).

Near $x=4$: The graph shows unbounded behavior so there is a vertical asymptote at $x=4$.

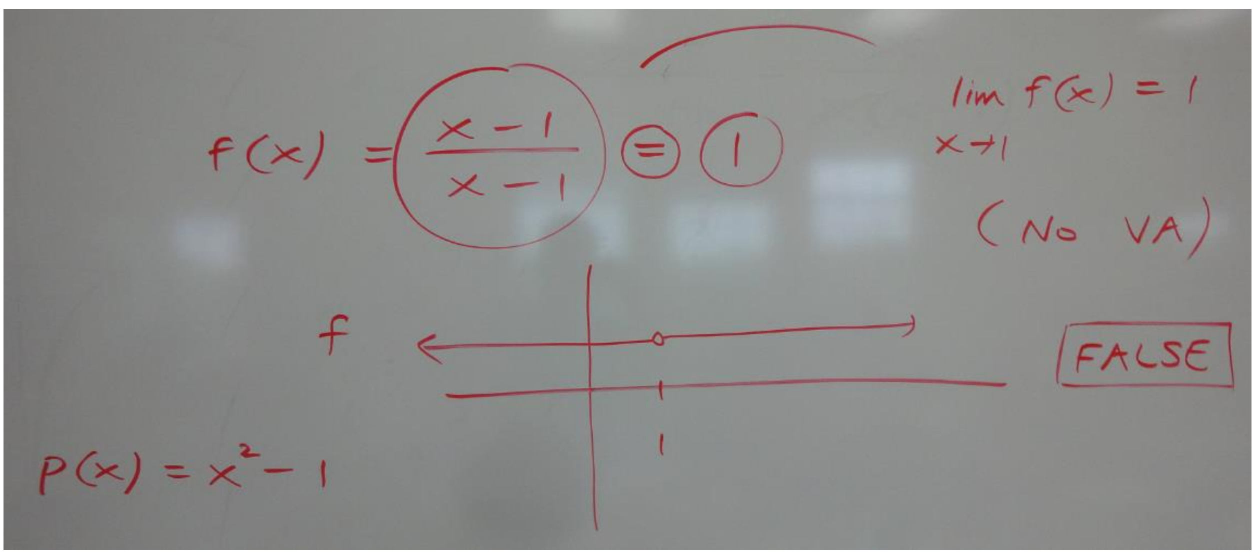
PROBLEM

TRUE / FALSE :

If $p(x)$ is a polynomial, then the function $f(x)=\dfrac{p(x)}{x-1}$ has a vertical asymptote at $x=1$.

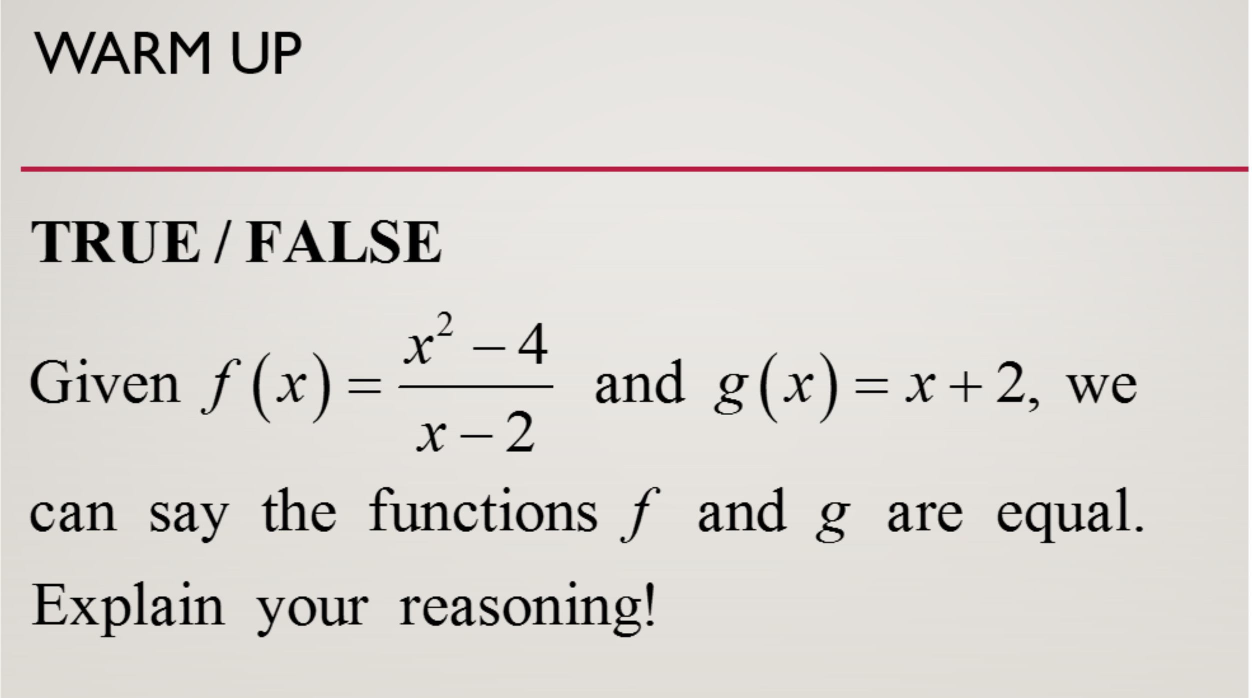
JUSTIFY!!

<u>This is false</u>!!! If you pull a $p(x)$ out of thin air, $f(x)=\dfrac{p(x)}{x-1}$ will *most likely* have a vertical asymptote at $x=1$ (so $\lim_{x\to 1} f(x)$ will not exist). However, there are many choices for $p(x)$ where $\lim_{x\to 1} f(x)$ will exist. See picture below, with $p(x)=x-1$:

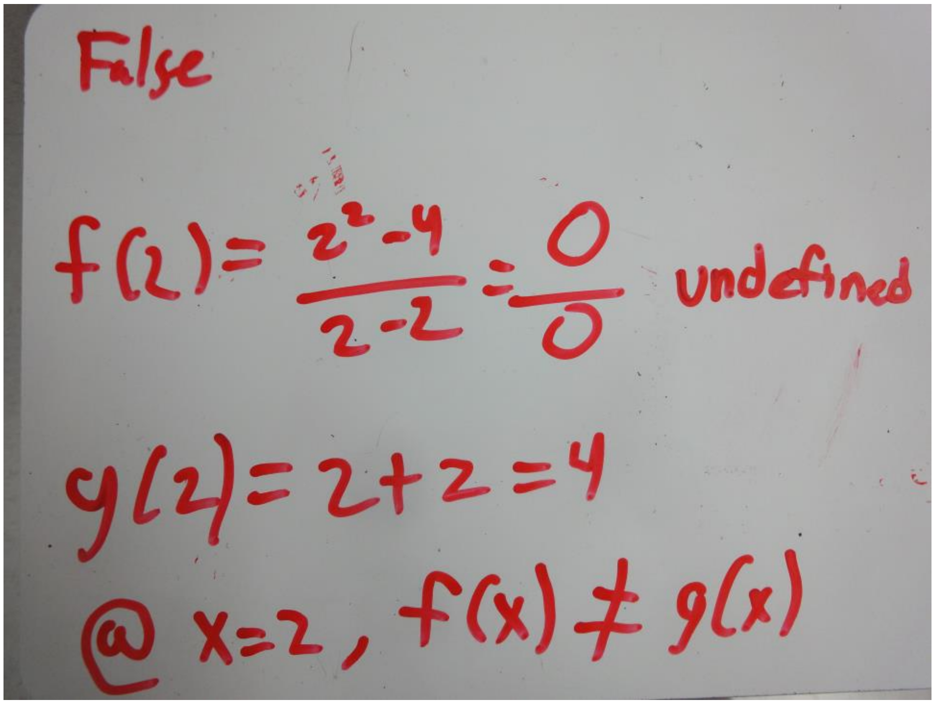


In fact, any $p(x)$ for which $x-1$ is a factor is enough to show that this statement is false.

1.6A: Limit Laws (Finding Limits Analytically)



This board sums it up well. In order for two functions to be equal, they must have identical outputs for each input.



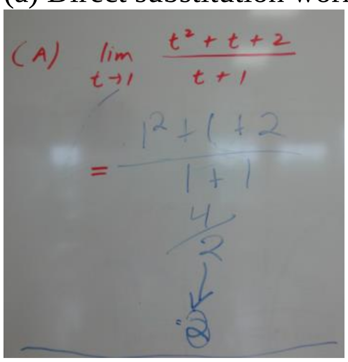
PROBLEM

Attempt paper/pencil methods to determine these limits.

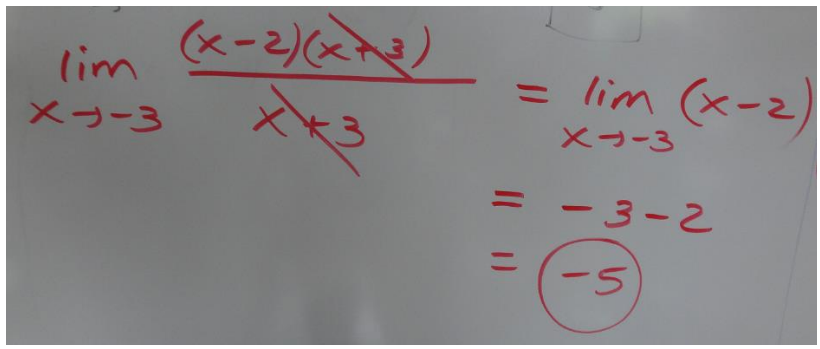
(a) $\lim_{t\to 1} \dfrac{t^2+t+2}{t+1}$ (c) $\lim_{r\to 0} \dfrac{\sqrt{r+1}-1}{r}$

(b) $\lim_{x\to -3} \dfrac{x^2+x-6}{x+3}$ (d) $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x}$

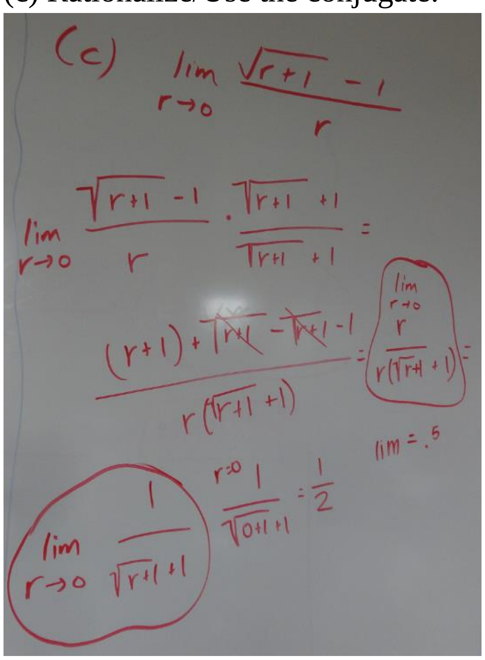
(a) Direct substitution works here:



(b) Factor/cancel to determine limit:



(c) Rationalize/Use the conjugate:



(d) Manipulate algebraically:

$$\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim_{x\to -4} \frac{\frac{1}{4}\frac{x}{x}+\frac{1}{x}\frac{4}{4}}{4+x}$$

$$= \lim_{x\to -4} \frac{\frac{x+4}{4x}}{4+x}$$

$$= \lim_{x\to -4} \frac{\cancel{x+4}}{4x}\cdot\frac{1}{\cancel{4+x}}$$

$$= \lim_{x\to -4} \frac{1}{4x}$$

$$= \frac{1}{4\cdot(-4)}$$

$$= \boxed{-\frac{1}{16}}$$

<u>IMPORTANT</u>: Problems (b)-(d) all begin with the 0/0 form so they need some kind of manipulation (factoring, multiplying by a conjugate, getting common denominators, rewriting, etc.) to reveal the limit. Note that each of these can be checked on a calculator by using the TABLE feature.

PROBLEM

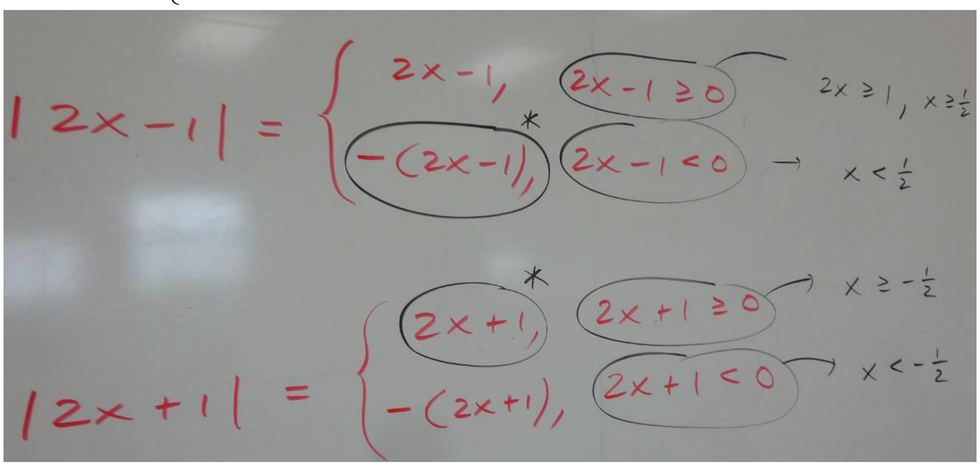
Consider the following: $\lim_{x\to 0}\frac{|2x-1|-|2x+1|}{x}$

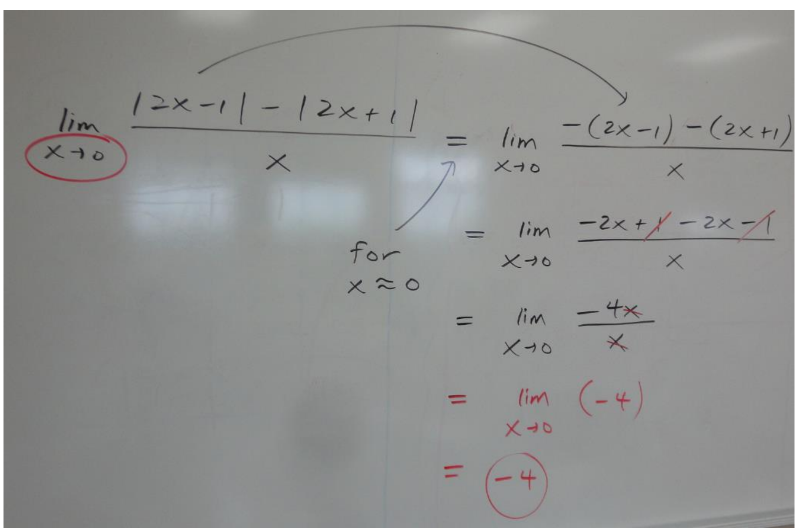
(a) Explain why evaluation of this limit is not straightforward.

(b) Find the limit by any means. (Try to find the limit analytically.)

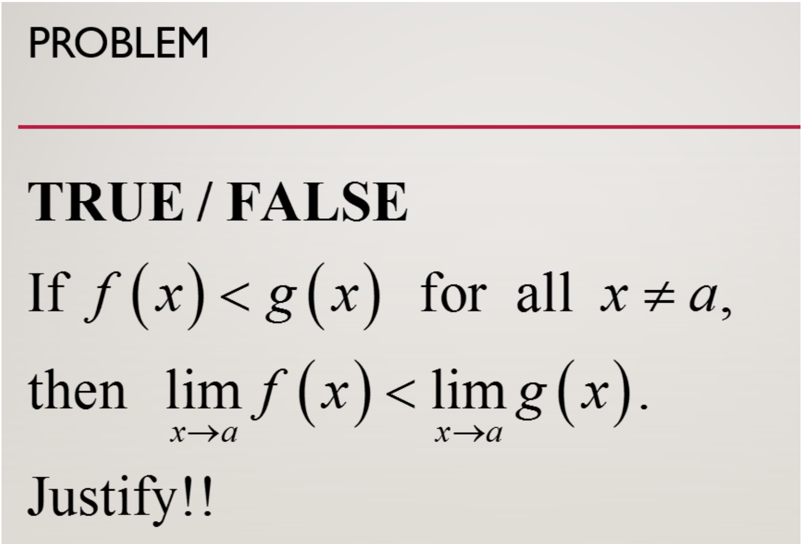
A graphing calculator will tell us $L=-4$ (table or graph). To solve this problem analytically, use what we know about absolute value. That is,

$$|x|=\begin{cases} x, & x\ge 0 \\ -x, & x<0 \end{cases}.$$

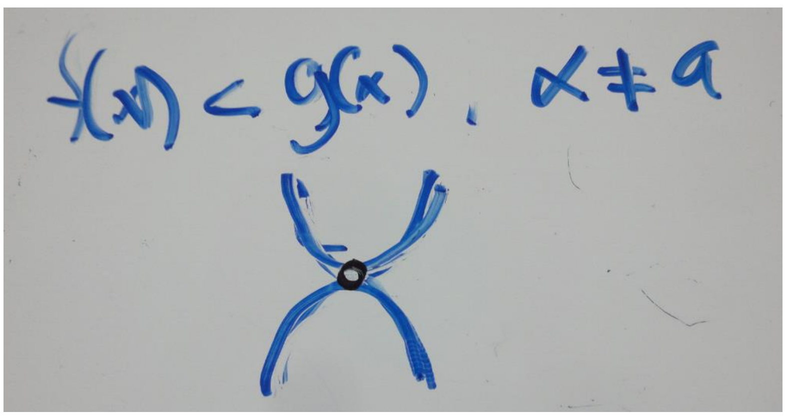


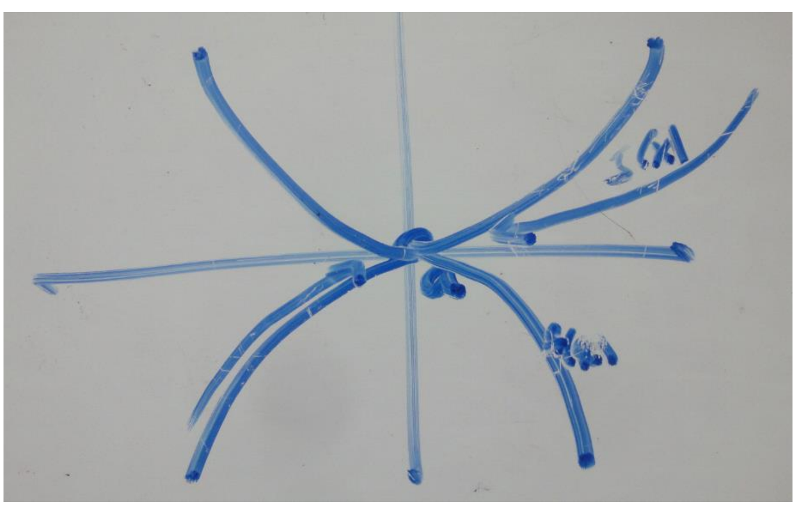


Lesson 1.6B: The Squeeze Theorem



These two boards capture the same idea (it is FALSE). It could be that $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$:





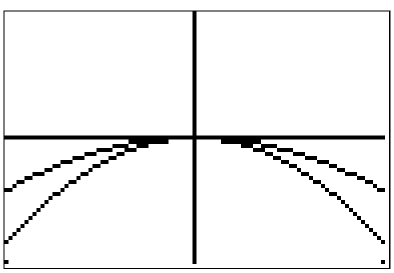
PROBLEM

Graph the three functions

$$f_1(x) = 1 - \frac{x^2}{6},\ f_2(x) = \frac{x \sin x}{2 - 2\cos x},\ f_3(x) = 1$$

on your calculator. Can you determine the value of $\lim_{x \to 0} f_2(x)$?

The quickest route here is to graph the functions and study what happens as $x \to 0$. The window below is [-1, 1, 0.8, 1.2].



From the graph, we can see that f_2 is squeezed between f_1 and f_3 (each approaches 1 for $x \to 0$). Based on this, $\lim_{x \to 0} f_2(x) = 1$.