May 24, 2017

1.8A: Continuity

PROBLEM

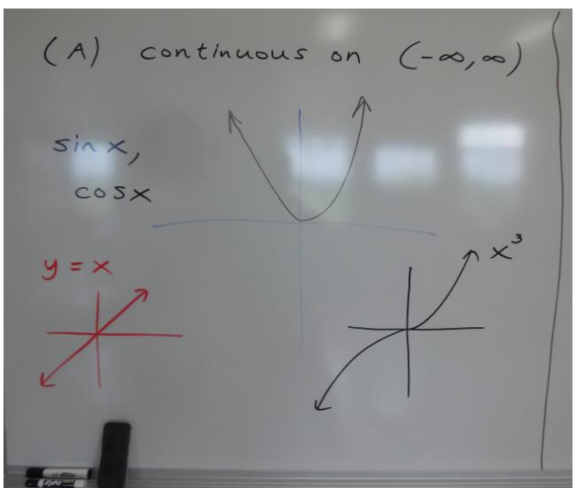
Give an example of a function (a formula or a graph is fine) $y = f(x)$ that is . . . .

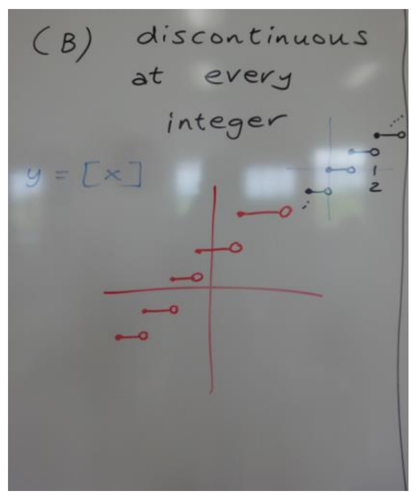
(a) continuous on $(-\infty, \infty)$.

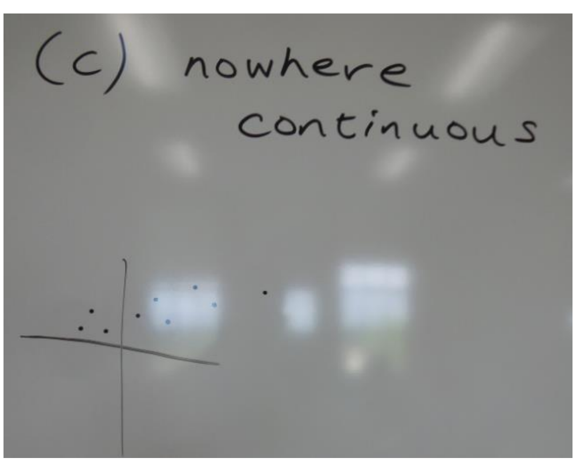
(b) discontinuous at every integer.

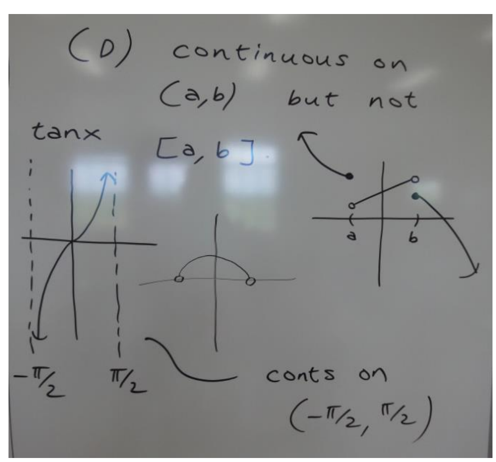
(c) nowhere continuous.

(d) continuous on (a, b) but not $[a, b]$.







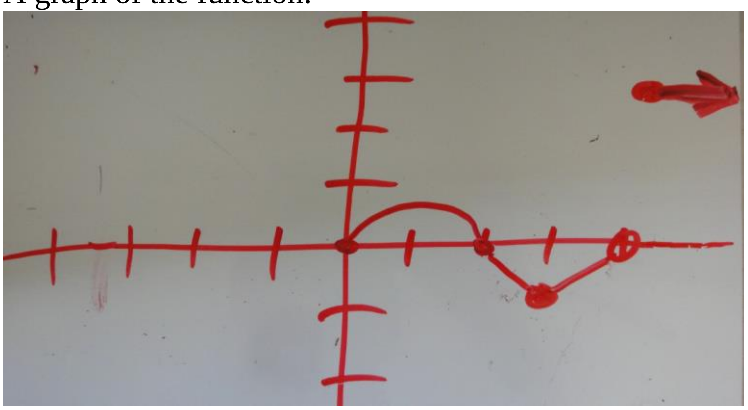


PROBLEM

Let $f(x)=\begin{cases} 2x-x^2 & \text{if } 0\le x\le 2 \\ 2-x & \text{if } 2<x\le 3 \\ x-4 & \text{if } 3<x<4 \\ \pi & \text{if } x\ge 4 \end{cases}$

Discuss the continuity of $f(x)$ at $x=2$, 3, and 4.

A graph of the function:



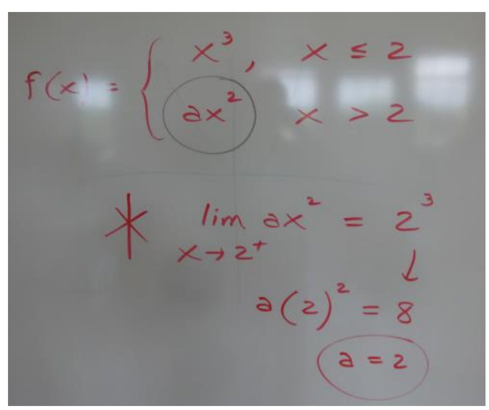
The function is continuous at $x=2$ and $x=3$ but not continuous at $x=4$ (notice that $\lim_{x\to 4} f(x)$ DNE). We can summarize by saying the function is continuous on $[0,4)\cup[4,\infty)$.

PROBLEM

Determine the value of a so that

$$f(x)=\begin{cases} x^3, & x\le 2 \\ ax^2, & x>2 \end{cases}$$

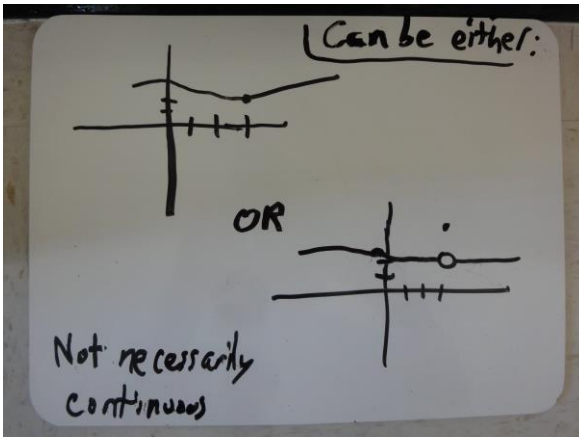
is continuous on the real number line.

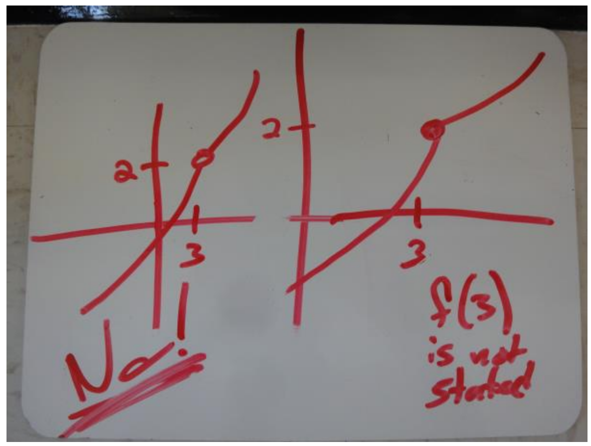


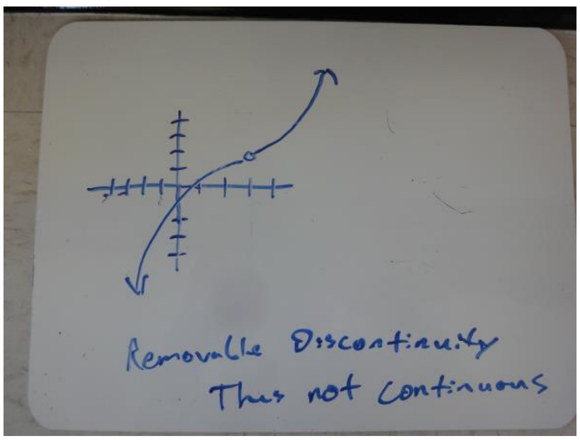
PROBLEM

Sketch the graph of $y = f(x)$ such that $\lim_{x \to 3^-} f(x) = 2$ and $\lim_{x \to 3^+} f(x) = 2.$

Is f continuous at $x = 3$? Explain.

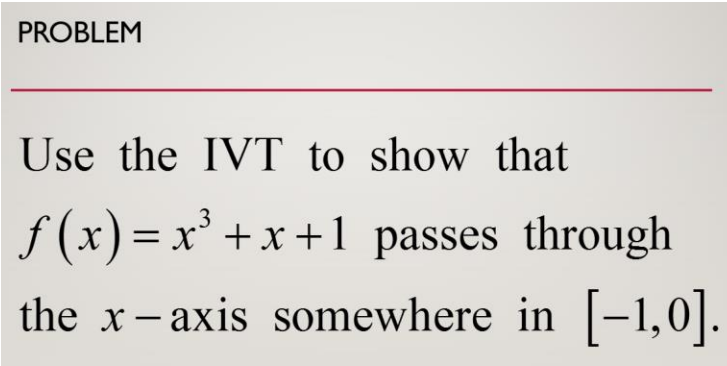


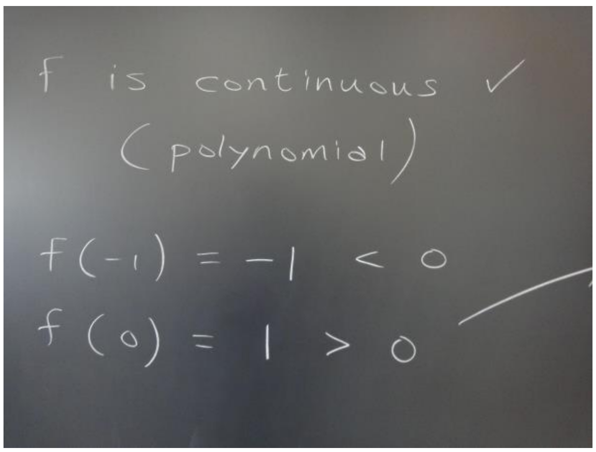


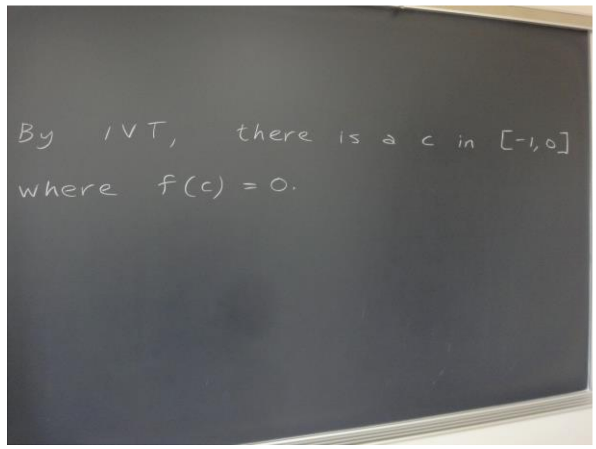


In general, the answer is no. Nothing is said about $f(3)$ so we know nothing about its existence. The function need not be continuous at $x = 3$ even though $\lim_{x \to 3} f(x) = 2$.

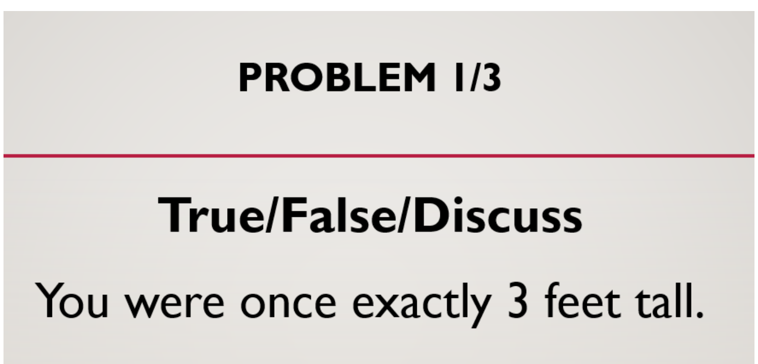
1.8B: Intermediate Value Theorem (IVT)



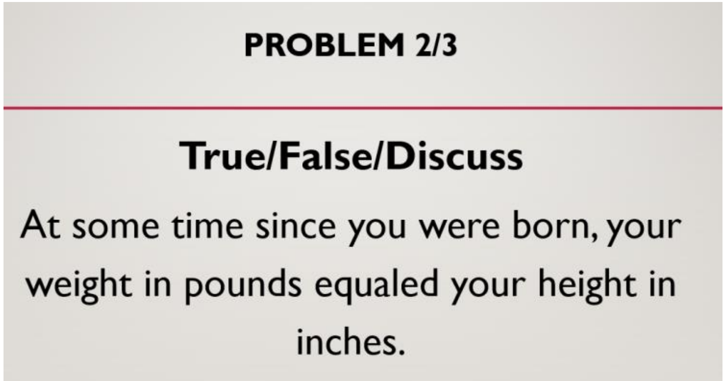


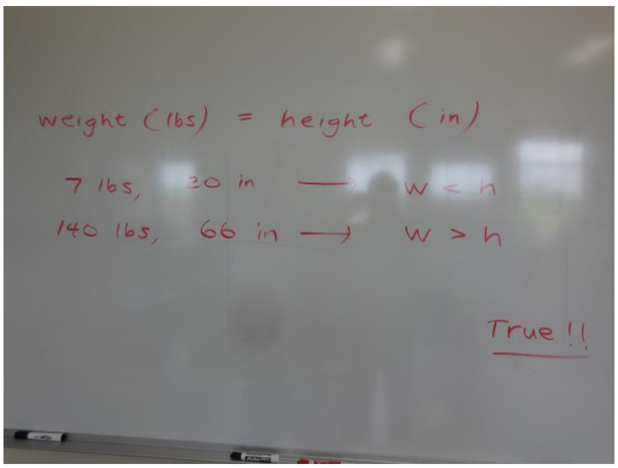


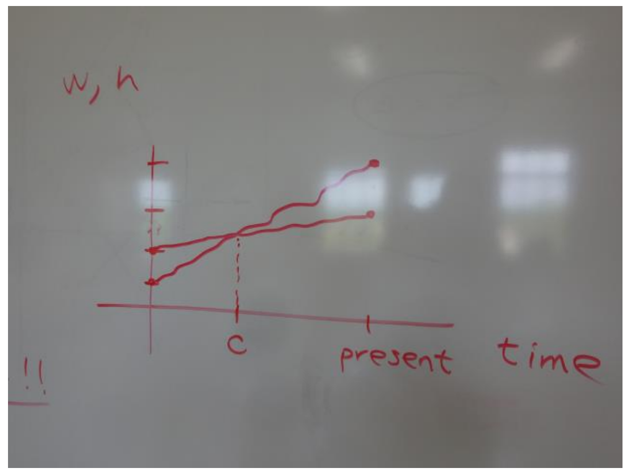
You could support this further by graphing f on the graphing calculator.

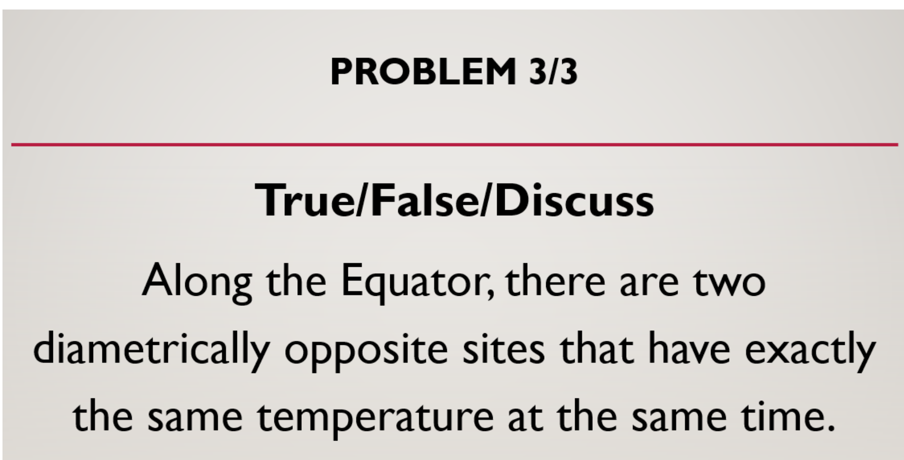


We discussed this in class. Everyone in the room was born under three feet tall but today, everyone is taller than three feet. Since growth is continuous, we were all—*at some point*—exactly three feet tall. We don't know exactly when this was and it was probably a different time in each of our lives but it certainly happened (IVT does not tell you when this happened but it guarantees that it happened).



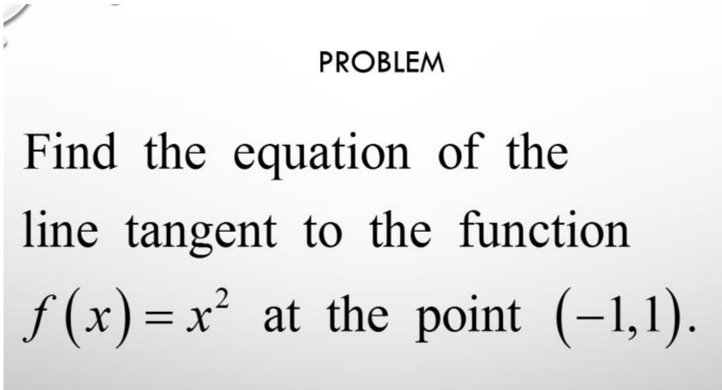




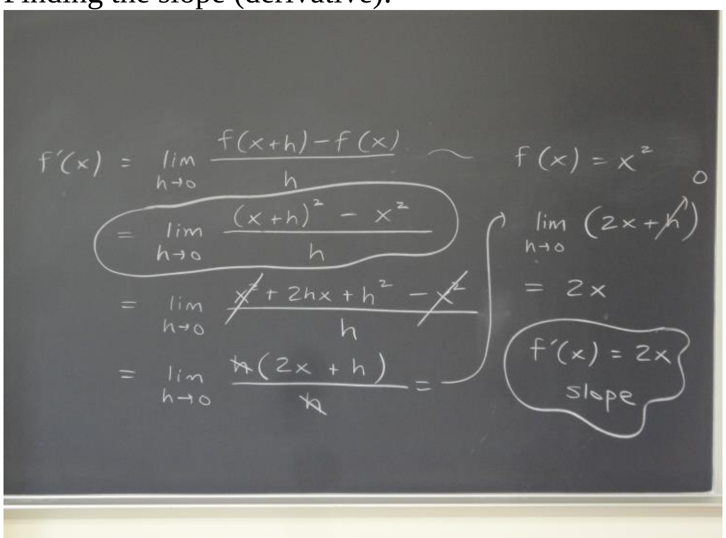


Also discussed in class. Take two temperature readings on the equator (they are most likely different). The important thing is that temperature is continuous so every temperature between the two readings is guaranteed along both arcs of the equator. You can then construct segments passing through the center of the earth with endpoints on the equator. At least one of these segments is guaranteed to have equal temperature readings at the endpoints. It's true!!

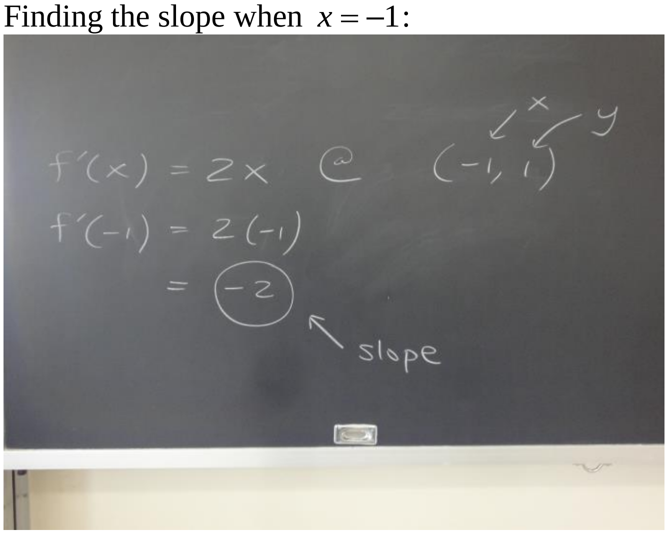
2.1: Rate of Change (Intro to the Derivative)



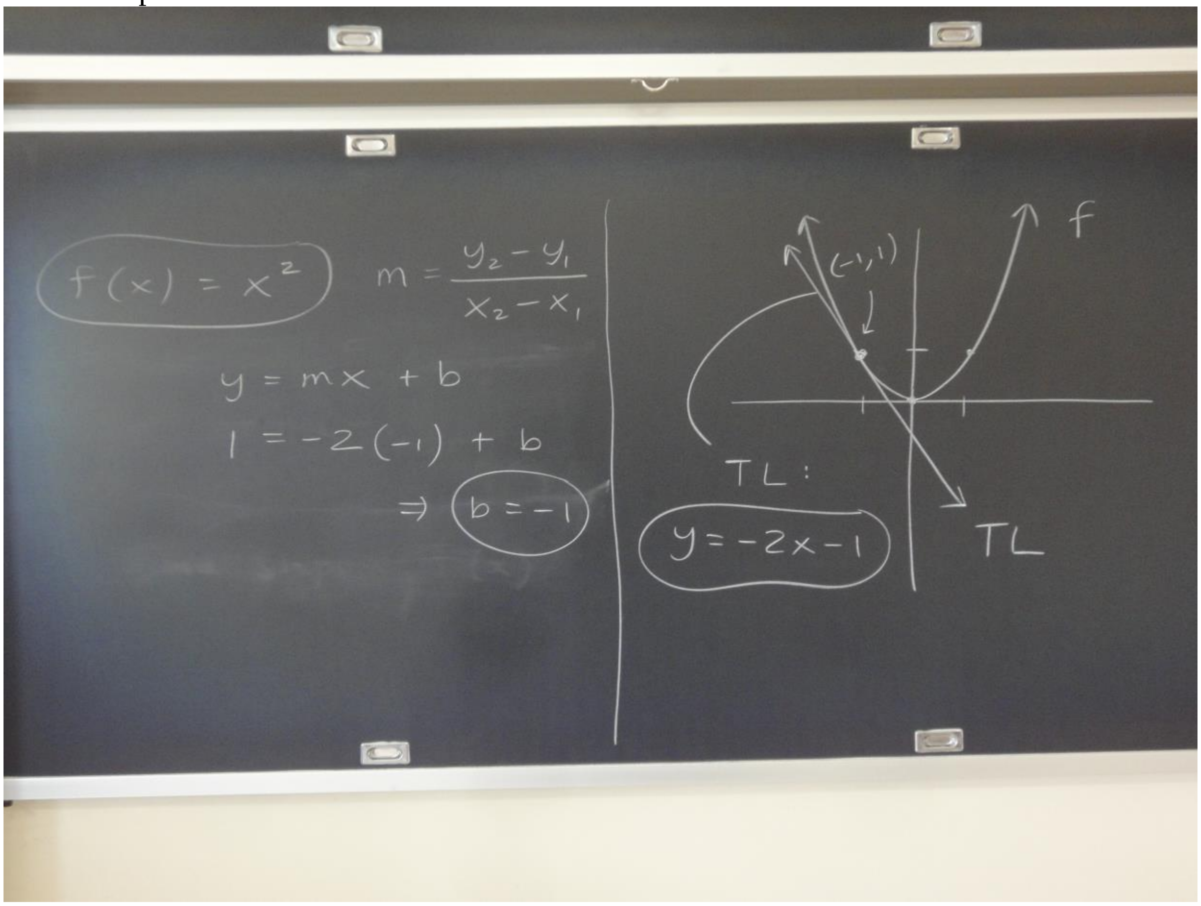
Finding the slope (derivative):

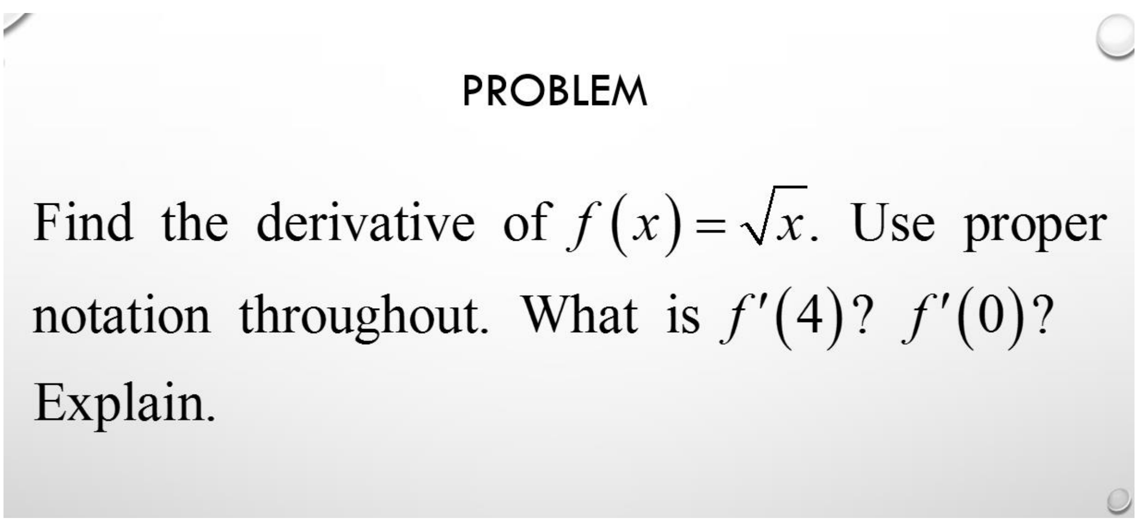


Finding the slope when $x = -1$:

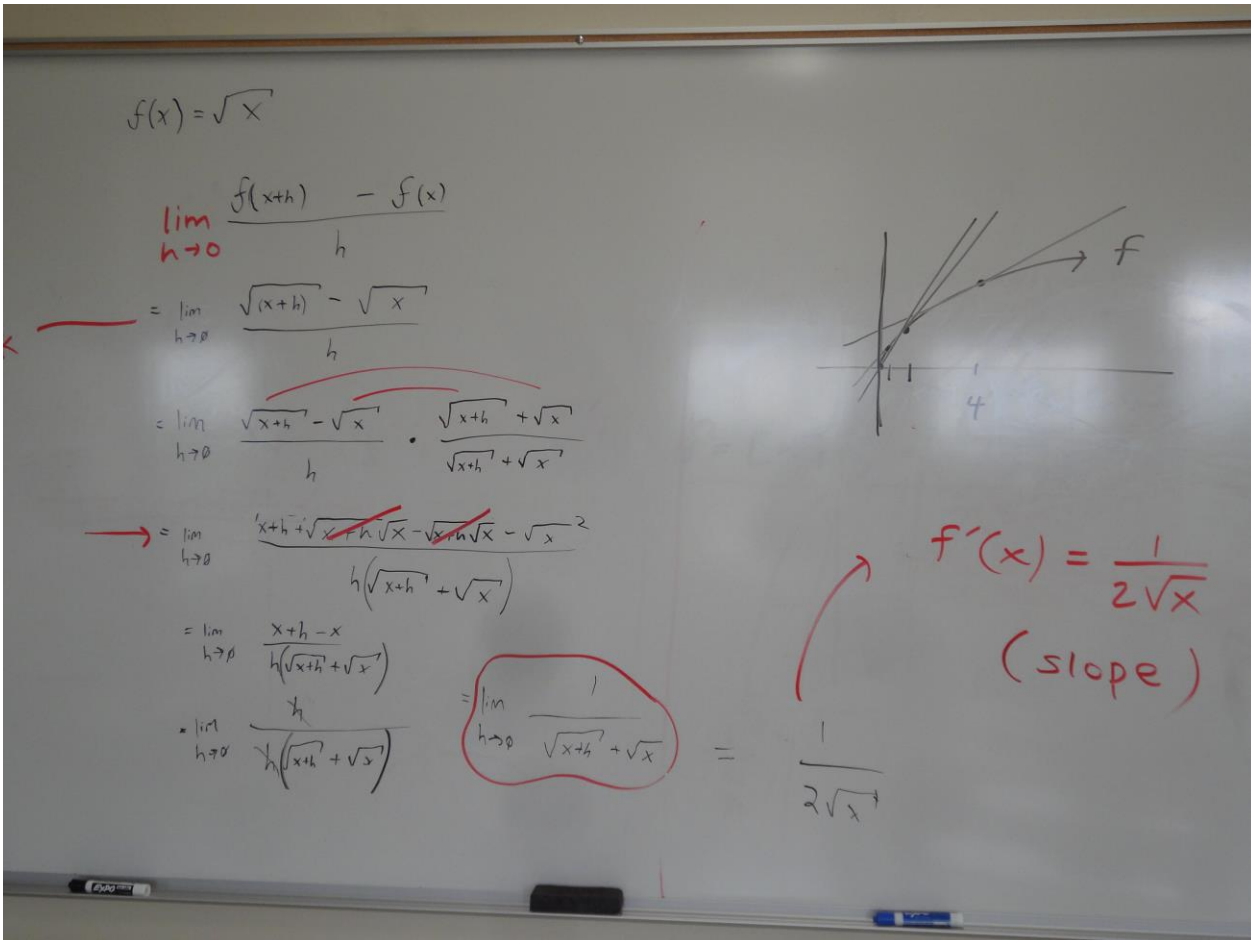


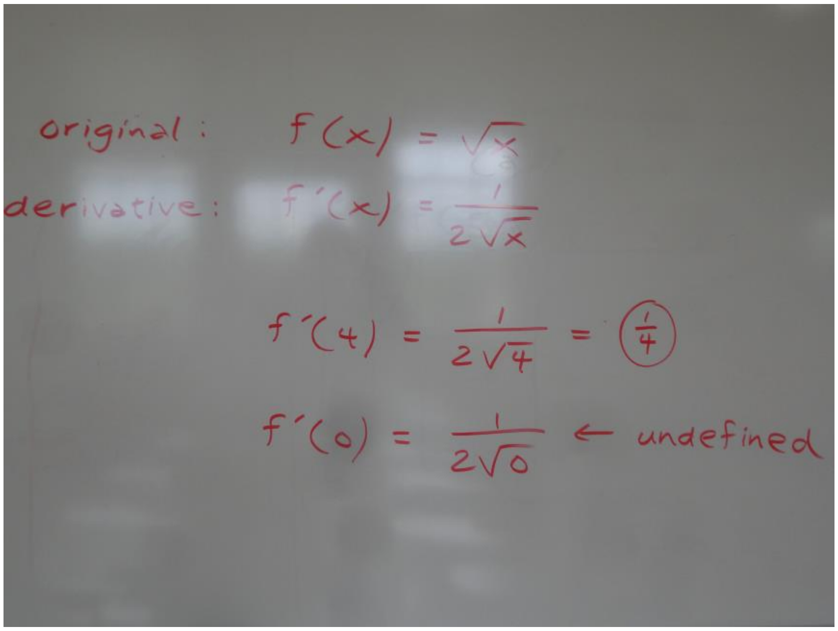
Finish the problem:





Finding $f'(x)$:


f(x) = √x
f′(x) = 1/(2√x)
(slope)
4
f


original: f(x) = √x
derivative: f′(x) = 1/(2√x)
f′(4) = 1/(2√4) = 1/4
f′(0) = 1/(2√0) ← undefined