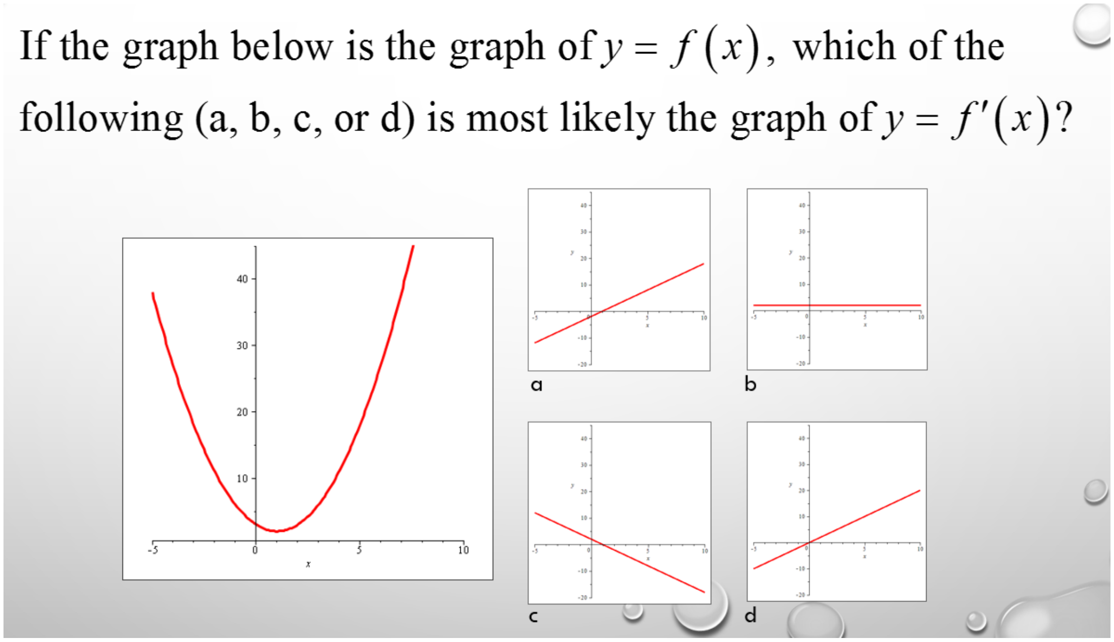
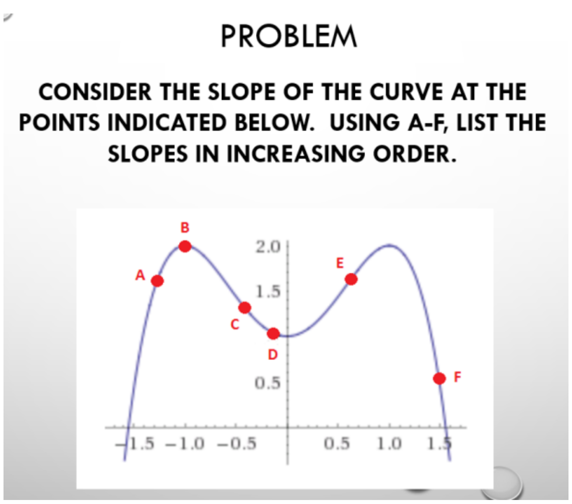
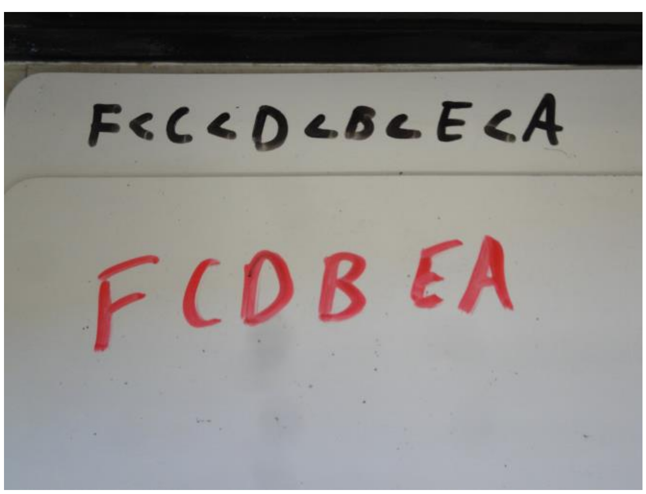
May 25, 2017

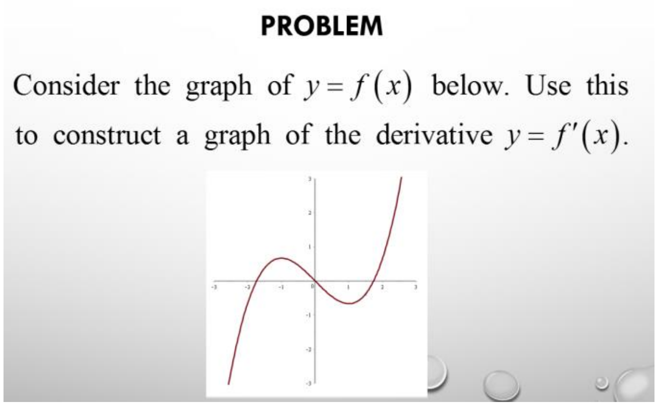
2.2: Derivative as a Function



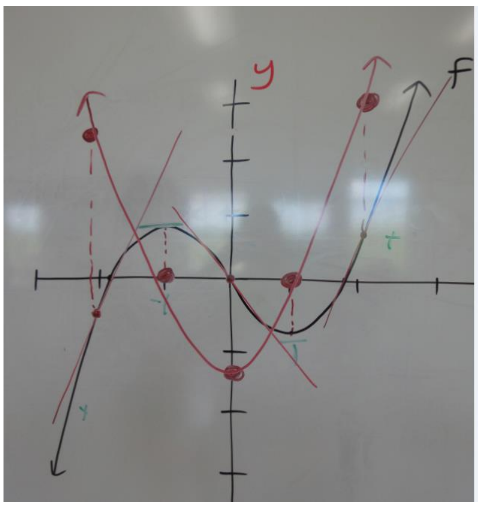
Explanation: The graph of $y = f'(x)$ is a graph of the **slopes** of $f(x)$. If you think of the tangent line slopes to $y = f(x)$, they start negative, eventually hit zero, and then become positive. From this, only (a) and (d) are possibilities (notice the outputs of (a) and (d) start below the *x*-axis, eventually hit zero, and then become positive). The original graph has a tangent line slope of zero just to the right of $x = 0$ (notice the graph of (a) crosses the *x*-axis just to the right of $x = 0$). The best answer is (a).

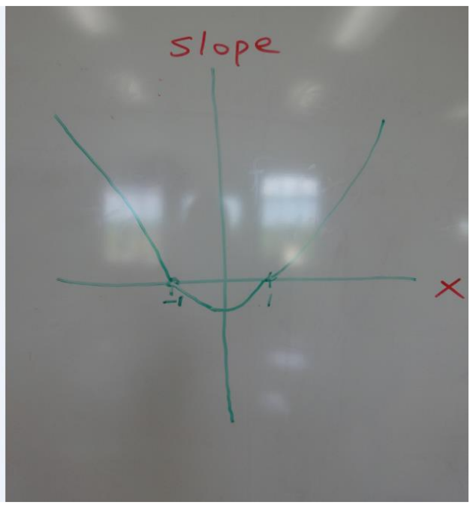


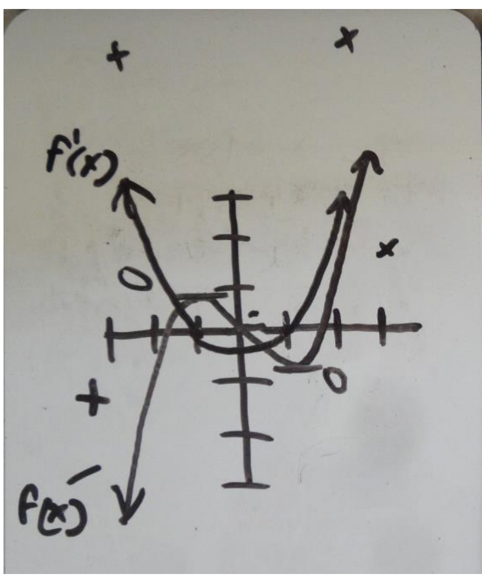


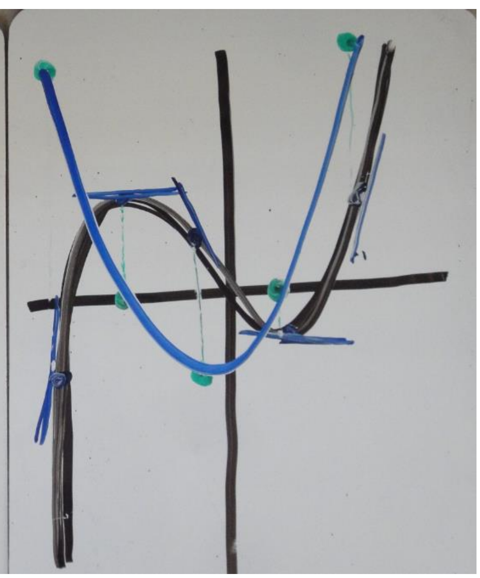


Much like the opening problem, it is helpful to visualize some tangent lines to the above graph and then make a plot of the values from left to right. Notice that although the below graphs are not all identical, they all have the same basic shape. That's the important part!!







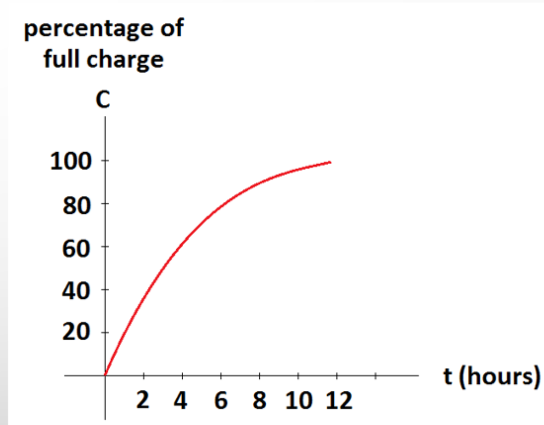


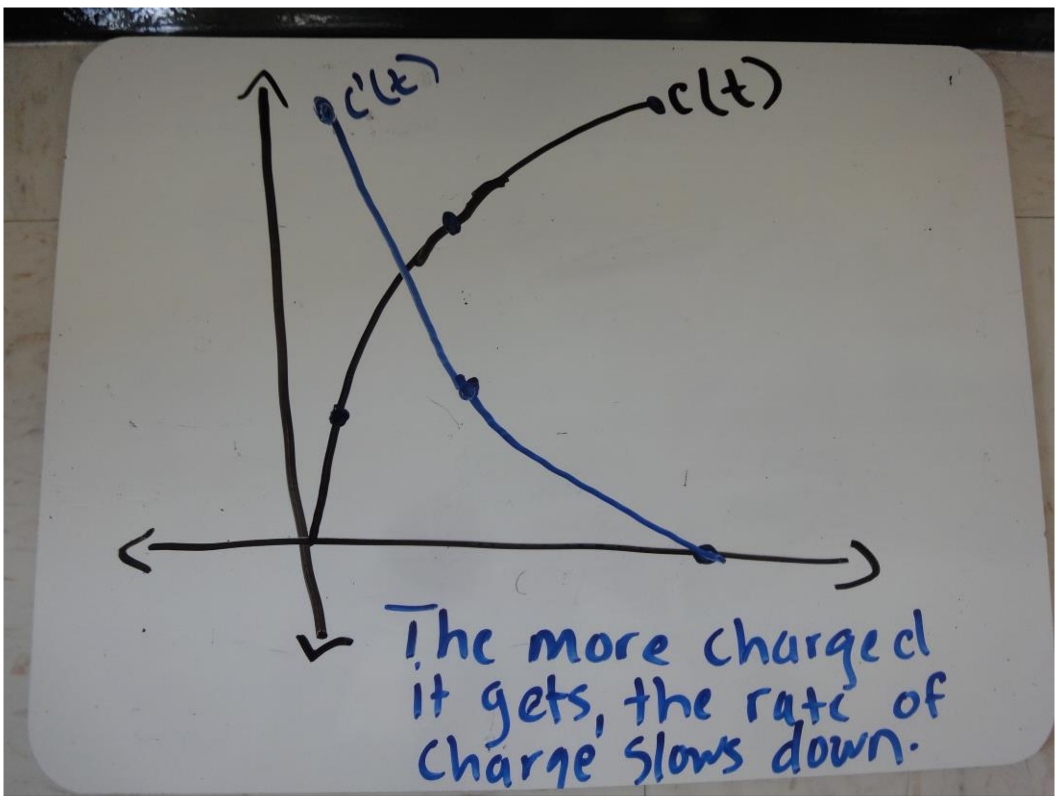
PROBLEM

A rechargeable battery is plugged into its charger. The graph shows $C(t)$, the percentage of full capacity that the battery reaches as a function of time t elapsed (in hours).

(a) What is the meaning of $C'(t)$?

(b) Sketch the graph of $C'(t)$. Interpret.





$C'(t)$ is the rate at which the battery charges. Notice that although $C(t)$ continues to climb (the battery gets more and more juice), the *rate* at which this happens decreases as the device approaches full charge. You can see this if you look at the graph of $C'(t)$ above (it is decreasing from left to right).

2.3A: Derivative Rules (Constant, Power, Constant Multiple, Sum/Difference)

FUNDAMENTALS

1. Constant Rule: $\frac{d}{dx}(c) = 0$
2. Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$
3. Constant Multiple Rule: $\frac{d}{dx}(cf(x)) = cf'(x)$
4. Sum/Difference Rule: $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$

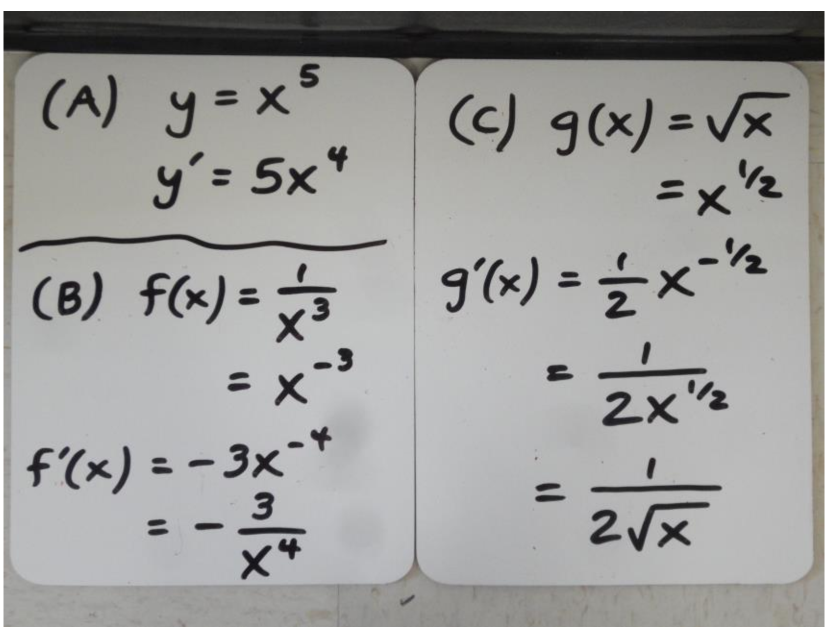
PROBLEM (LEVEL I)

Find the derivatives. Use appropriate notation.

(a) $y = x^5$

(b) $f(x) = \frac{1}{x^3}$

(c) $g(x) = \sqrt{x}$



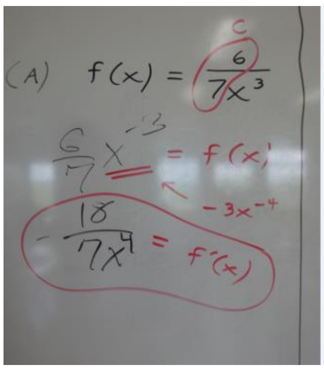
PROBLEM (LEVEL II)

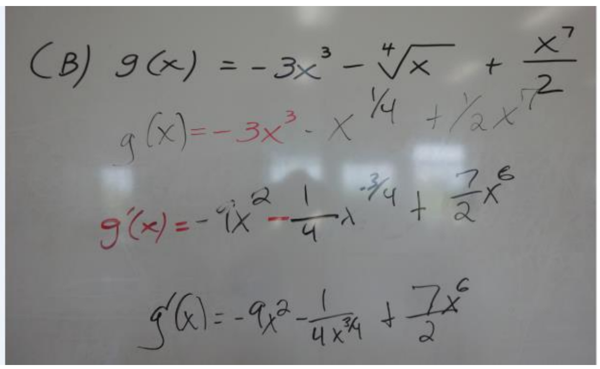
Find the derivatives. Use appropriate notation.

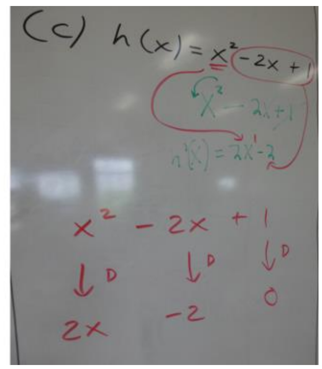
(a) $f(x) = \dfrac{6}{7x^3}$

(b) $g(x) = -3x^3 - \sqrt[4]{x} + \dfrac{x^7}{2}$

(c) $h(x) = x^2 - 2x + 1$







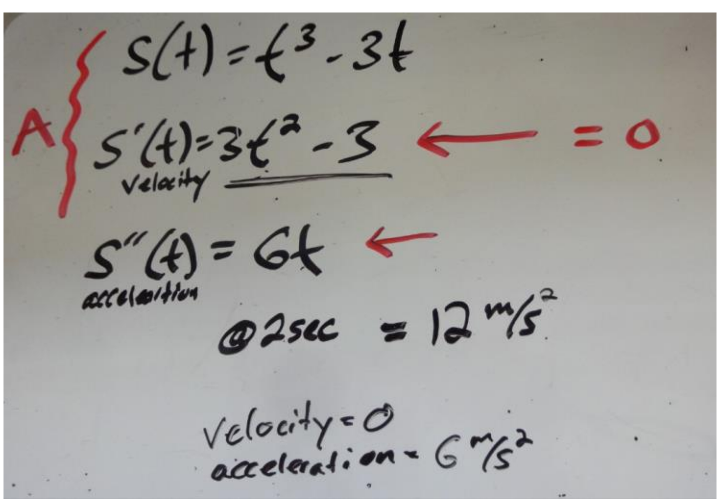
PROBLEM

The equation of motion of a particle is $s = t^3 - 3t$, where s is in meters and t is in seconds. Find

(a) the velocity and acceleration as functions of t,

(b) the acceleration after 2 sec,

(c) the acceleration when the velocity is 0.



PROBLEM

True or false? Explain why.

(a) $\frac{d}{dx}\left(e^{7}\right) = 7e^{6}$ where $e = 2.71828...$

(b) $\frac{d}{dx}\left(\frac{1}{x^{3}}\right) = \frac{1}{3x^{2}}$ (c) $\frac{d}{dx}\left(\frac{x}{\pi}\right) = \frac{1}{\pi}$

(d) If $f'(x) = g'(x)$, then $f(x) = g(x)$.

1. false, constant
2. false, $-3x^{-4}$
3. True
4. false $f(x) = x^2 + 10$
   $g(x) = x^2$

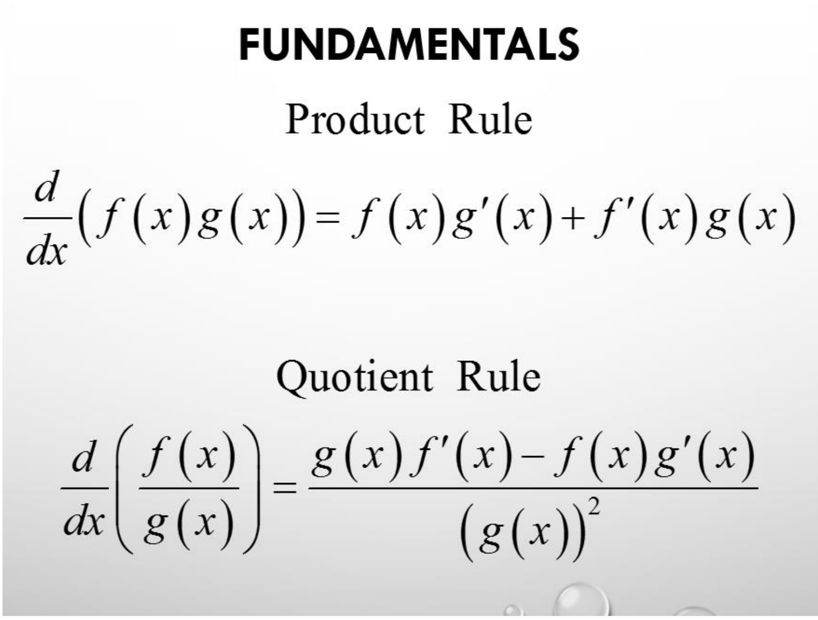
A: False e is a constant

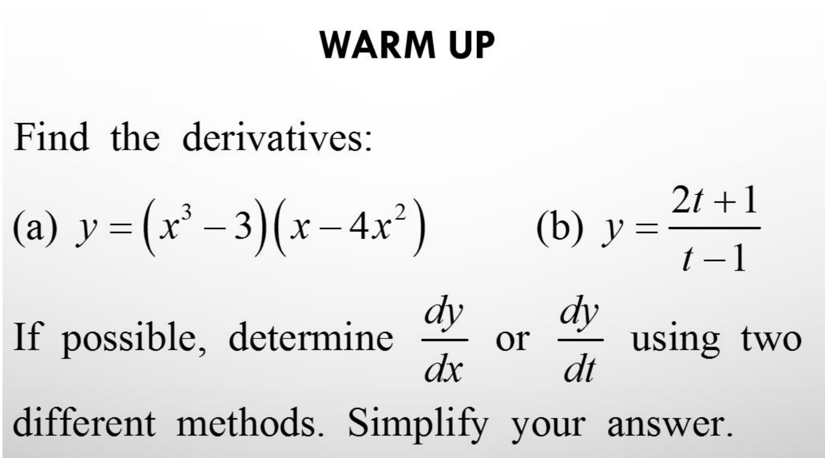
B: False x^3 cannot be derived in denominator

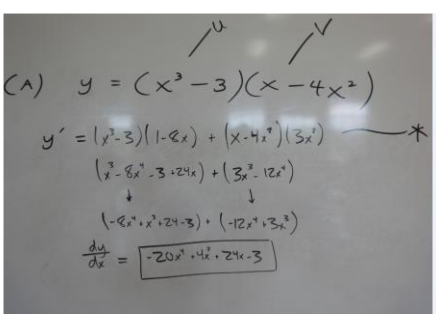
C: True

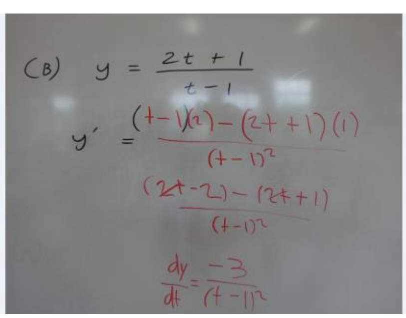
D: False $f(x) = g(x)$ can be different but still get a same $f'(x) = g'(x)$

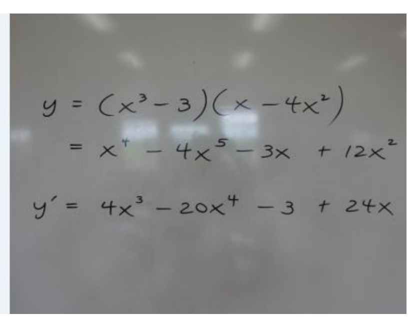
2.3B: More Derivative Rules (Product + Quotient Rules)











Notice there are two different ways we can approach problem (a)—the calculus first method (seen all the way on the left) or the algebra first method (seen all the way to the right).