May 26, 2017

2.4: Trigonometry & Derivatives

TWO IMPORTANT LIMITS

1. $\lim\limits_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1$

2. $\lim\limits_{\theta \to 0} \dfrac{\cos\theta - 1}{\theta} = 0$

We proved that $\frac{d}{dx}(\sin x) = \cos x$. It uses both of the above limits:

$$
\begin{aligned}
\frac{d}{dx}(\sin x) &= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} \\
&= \lim_{h\to 0} \frac{\sin x \cos(h) + \cos x \sin(h) - \sin x}{h} \\
&= \lim_{h\to 0} \left[\sin x \left(\frac{\cos(h) - 1}{h} \right) + \cos x \left(\frac{\sin(h)}{h} \right) \right] \\
&= \sin x \cdot 0 + \cos x \cdot 1 \\
&= \cos x
\end{aligned}
$$

FUNDAMENTALS

1. $\dfrac{d}{dx}(\sin x) = \cos x$
2. $\dfrac{d}{dx}(\cos x) = -\sin x$
3. $\dfrac{d}{dx}(\tan x) = \sec^2 x$
4. $\dfrac{d}{dx}(\cot x) = -\csc^2 x$
5. $\dfrac{d}{dx}(\sec x) = \sec x \tan x$
6. $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$

PROBLEM

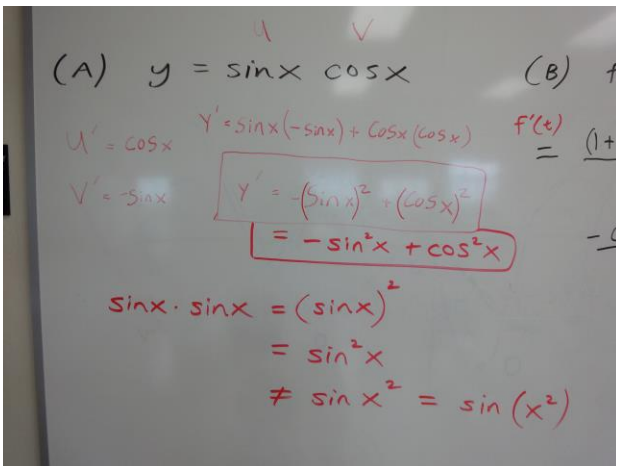
Find the derivatives; simplify to a reasonable point.

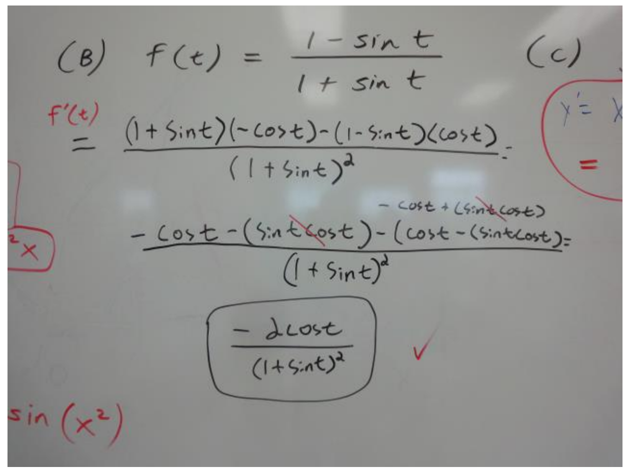
(a) $y = \sin x \cos x$

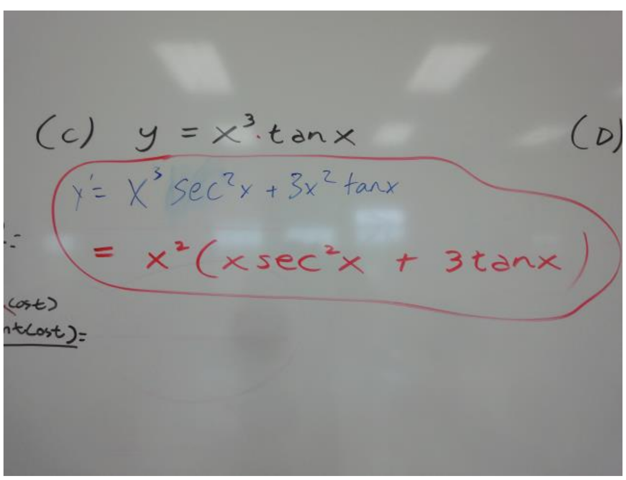
(b) $f(t) = \dfrac{1 - \sin t}{1 + \sin t}$

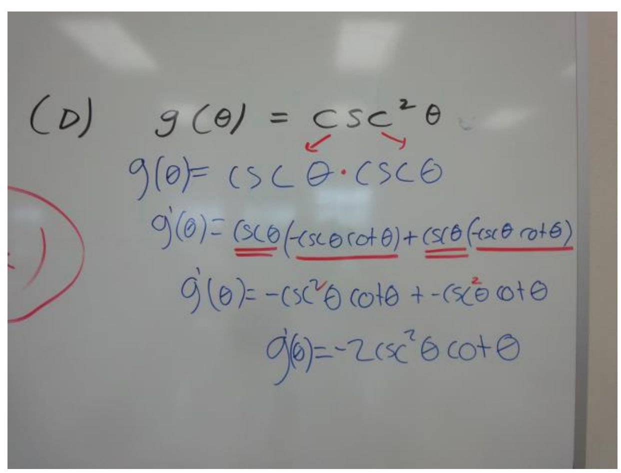
(c) $y = x^3 \tan x$

(d) $g(\theta) = \csc^2\theta$









PROBLEM

Use the Quotient Rule to prove that $\frac{d}{dx}(\sec x) = \sec x \tan x.$