

May 26, 2017

2.4: Trigonometry & Derivatives

TWO IMPORTANT LIMITS

$$1. \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$2. \lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0$$

We proved that $\frac{d}{dx}(\sin x) = \cos x$. It uses both of the above limits:

$$\begin{aligned} \frac{d}{dx}(\sin x) &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin x \cos(h) + \cos x \sin(h) - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \left[\sin x \left(\frac{\cos(h) - 1}{h} \right) + \cos x \left(\frac{\sin(h)}{h} \right) \right] \\ &= \sin x \cdot 0 + \cos x \cdot 1 \\ &= \cos x \end{aligned}$$

FUNDAMENTALS

$$1. \frac{d}{dx}(\sin x) = \cos x$$

$$2. \frac{d}{dx}(\cos x) = -\sin x$$

$$3. \frac{d}{dx}(\tan x) = \sec^2 x$$

$$4. \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$5. \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$6. \frac{d}{dx}(\csc x) = -\csc x \cot x$$

PROBLEM

Find the derivatives; simplify to a reasonable point.

(a) $y = \sin x \cos x$

(c) $y = x^3 \tan x$

(b) $f(t) = \frac{1 - \sin t}{1 + \sin t}$

(d) $g(\theta) = \csc^2 \theta$

(A) $y = \sin x \cos x$

$u' = \cos x$ $v' = -\sin x$

$Y' = \sin x(-\sin x) + \cos x(\cos x)$

$Y' = -(\sin x)^2 + (\cos x)^2$

$= -\sin^2 x + \cos^2 x$

$\sin x \cdot \sin x = (\sin x)^2$

$= \sin^2 x$

$\neq \sin x^2 = \sin(x^2)$

(B) $f(t) = \frac{1 - \sin t}{1 + \sin t}$

$f'(t) = \frac{(1 + \sin t)(-\cos t) - (1 - \sin t)(\cos t)}{(1 + \sin t)^2}$

$= \frac{-\cos t + \sin t \cos t - \cos t + \sin t \cos t}{(1 + \sin t)^2}$

$= \frac{-2\cos t}{(1 + \sin t)^2}$

(c) $y = x^3 \cdot \tan x$

$y' = x^3 \sec^2 x + 3x^2 \tan x$

$= x^2(x \sec^2 x + 3 \tan x)$

(D) $g(\theta) = \csc^2 \theta$

$g(\theta) = \csc \theta \cdot \csc \theta$

$g'(\theta) = \underline{\csc \theta}(-\csc \theta \cot \theta) + \underline{\csc \theta}(-\csc \theta \cot \theta)$

$g'(\theta) = -\csc^2 \theta \cot \theta + -\csc^2 \theta \cot \theta$

$g'(\theta) = -2 \csc^2 \theta \cot \theta$

PROBLEM

Use the Quotient Rule to prove

$$\text{that } \frac{d}{dx}(\sec x) = \sec x \tan x.$$

$$\begin{aligned} \frac{d}{dx}(\sec x) &= \sec x \tan x \\ &\quad \swarrow \text{(Quotient Rule)} \\ \frac{d}{dx}\left(\frac{1}{\cos x}\right) &= \frac{\cos x \cdot (0) - 1 \cdot (-\sin x)}{(\cos x)^2} \end{aligned}$$

$$\begin{aligned} \left(\frac{1}{\cos x}\right)' &= \frac{\sin x}{\cos^2 x} \\ &= \frac{\sin x}{\cos x \cdot \cos x} \\ &= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x \quad \square \end{aligned}$$