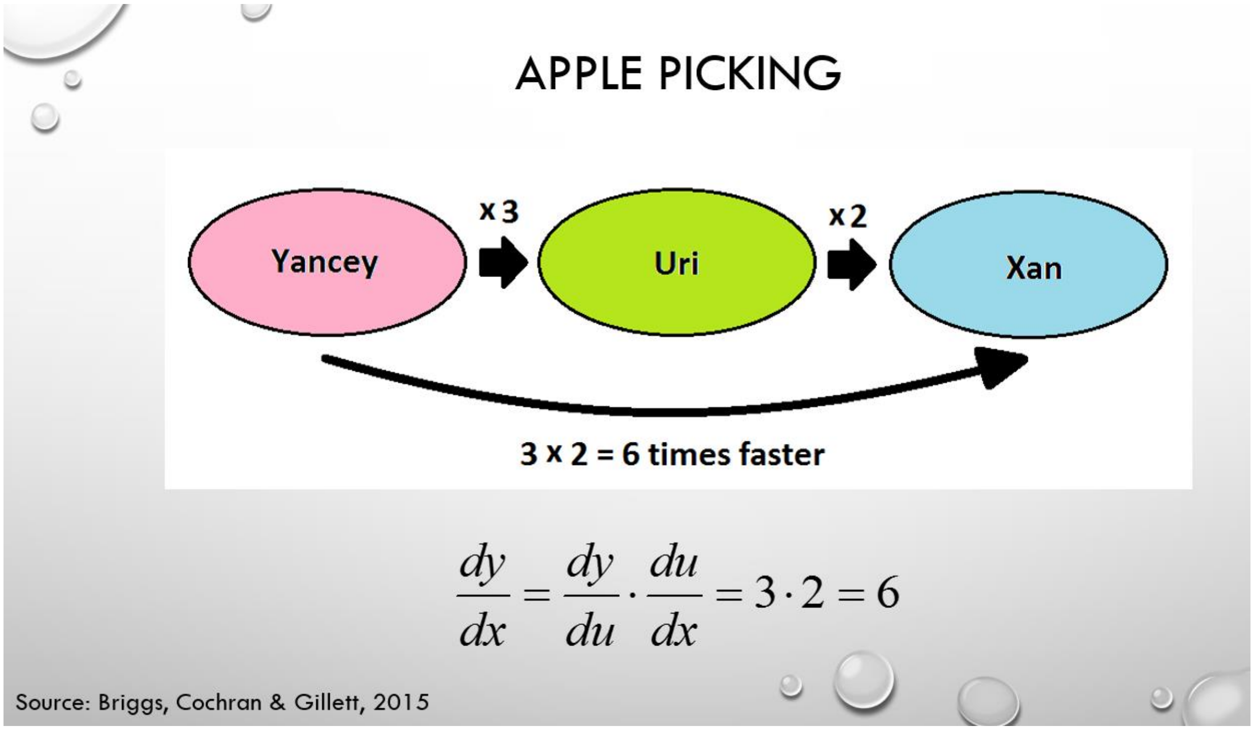
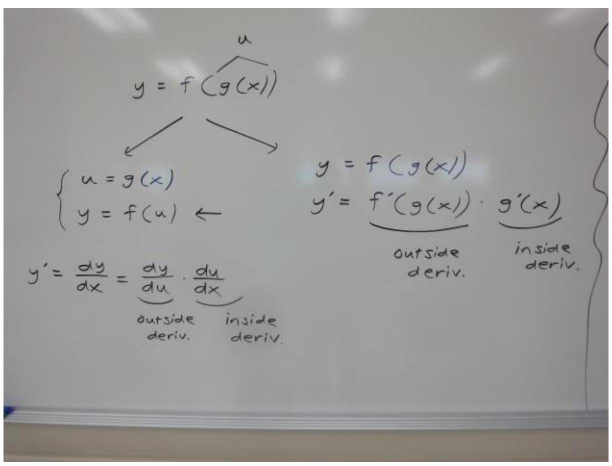
May 30, 2017

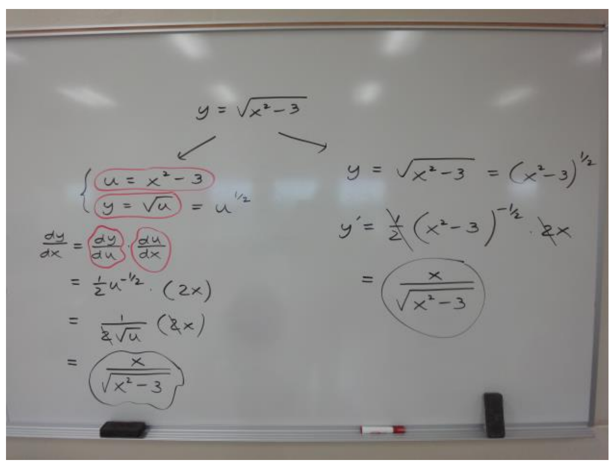
2.5: The Chain Rule



Two Perspectives:

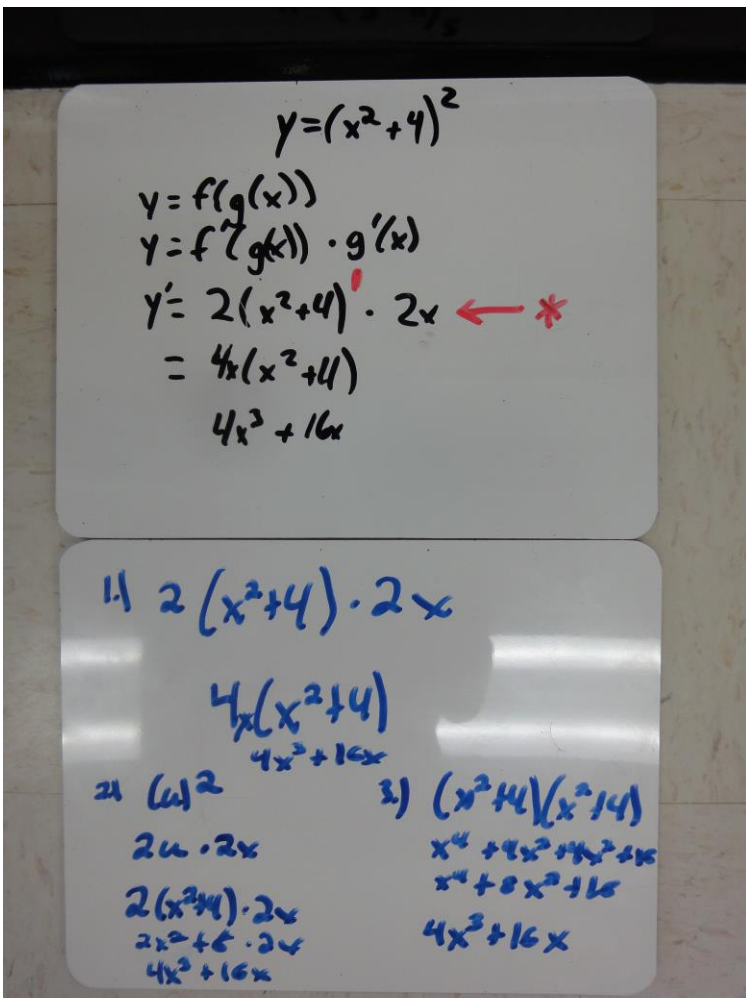


Sample Problem:



WARM UP

Consider the function $y=\left(x^2+4\right)^2$. You now have two different ways to differentiate this function. Show that the final answers agree.



Also worth mentioning: You can use the product rule on $y=\left(x^2+4\right)\left(x^2+4\right)$ so

$y'=\left(x^2+4\right)(2x)+(2x)\left(x^2+4\right)=2(2x)\left(x^2+4\right)=\boxed{4x^3+16x}$.

PROBLEM (LEVEL I)

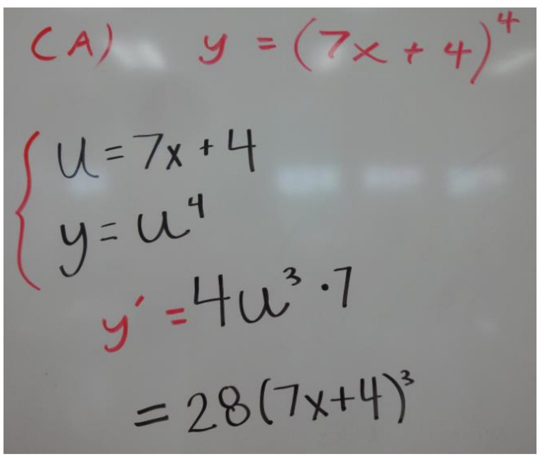
Find the derivative.

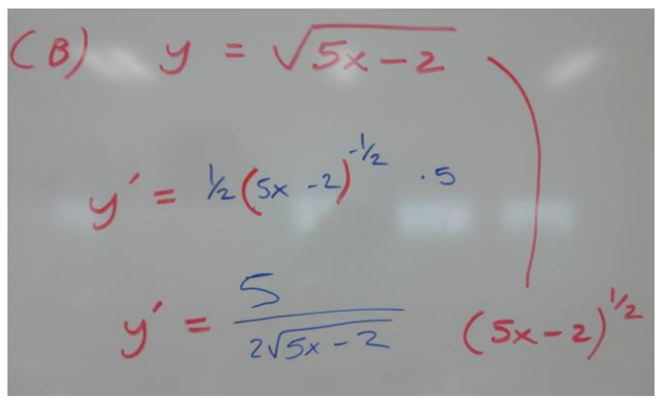
(a) $y = (7x+4)^4$

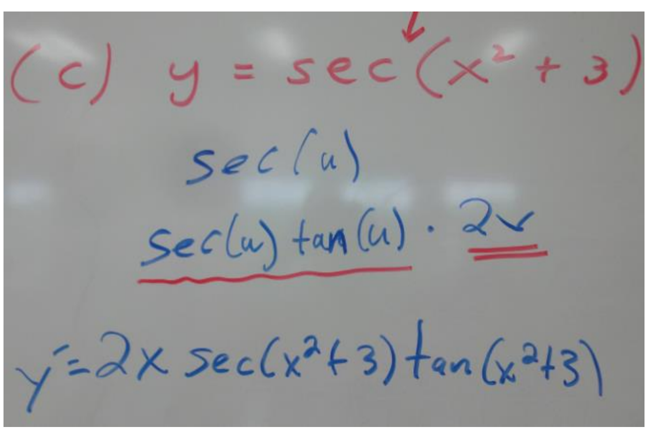
(c) $y = \sec\left(x^2+3\right)$

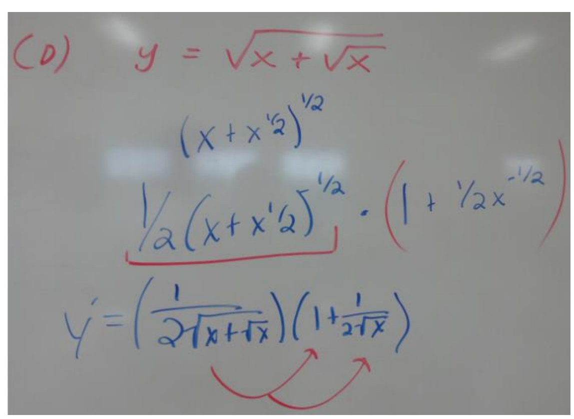
(b) $y = \sqrt{5x-2}$

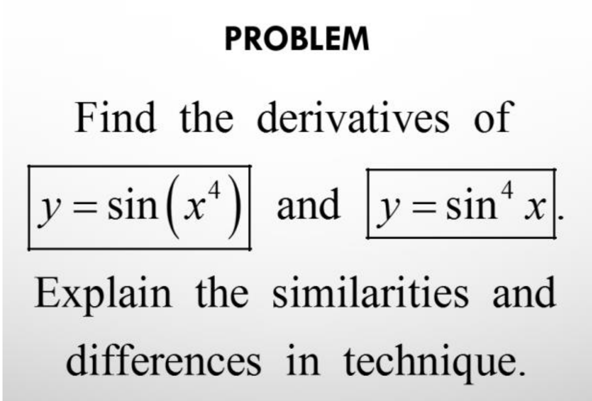
(d) $y = \sqrt{x+\sqrt{x}}$

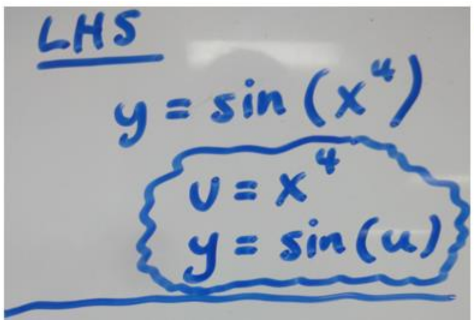


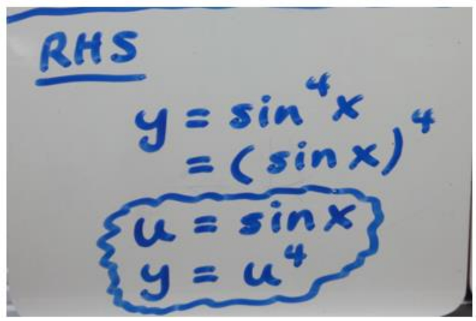




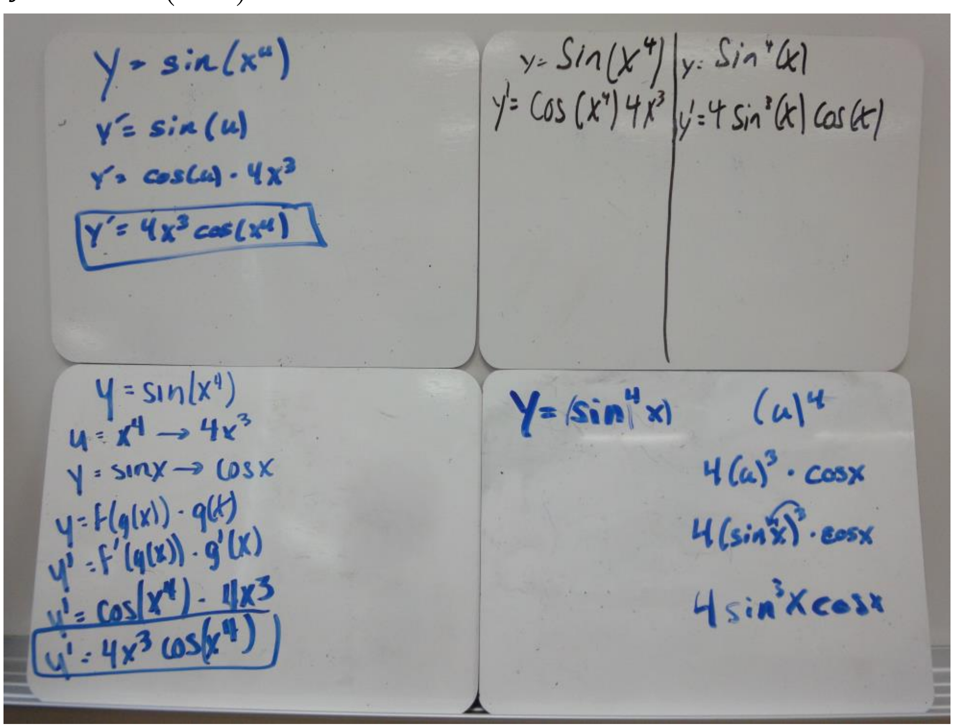








The above boards show that although the same two functions are at work here—the **sine** function and the **fourth power**—they play different roles in the two problems. In $y=\sin\left(x^4\right)$, the x^4 is the inside function while sine is the outside function. This is reversed in the problem $y=\sin^4 x=(\sin x)^4$. See below:



PROBLEM (LEVEL II)

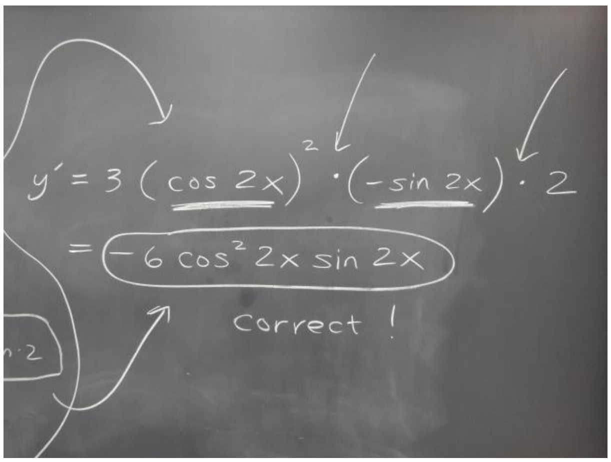
Find the derivative.

(a) $y=\cos^3(2x)$

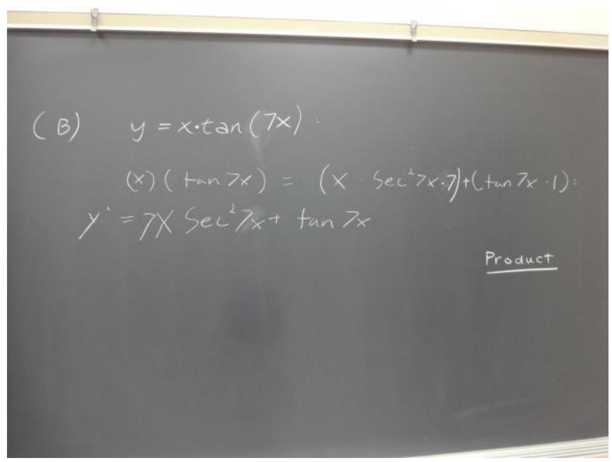
(b) $y=x\tan(7x)$

(c) $y=x^3\sqrt{1-x^2}$

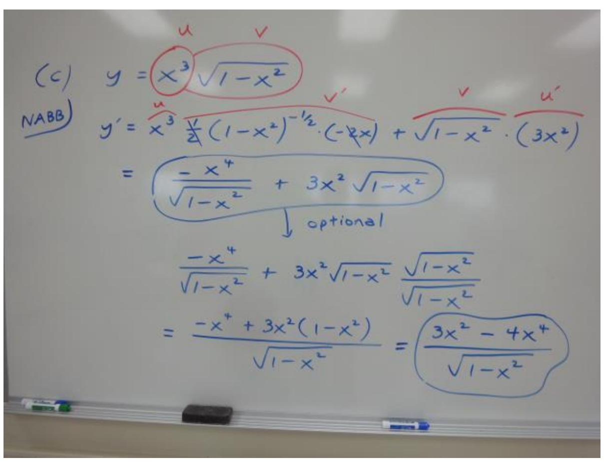
(d) $y=\left(\dfrac{\sin x}{x+2}\right)^2$



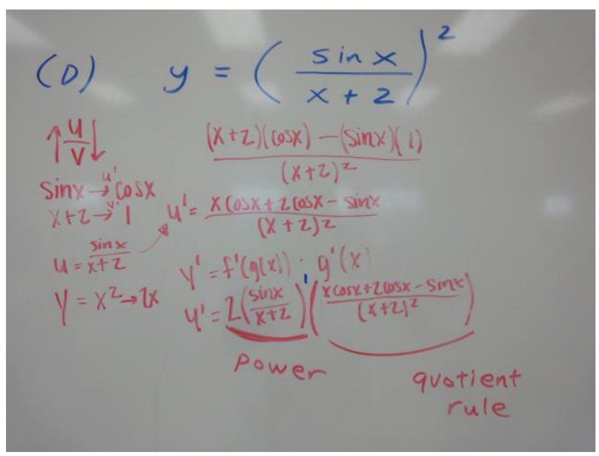
Notice **two** applications of the chain rule above!!



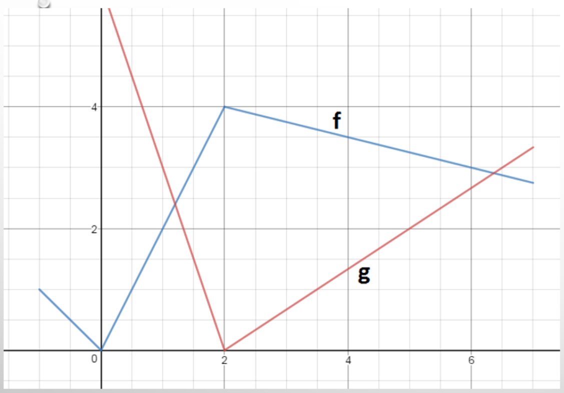
Product & Chain needed above.



Product & Chain needed above.



Power & Quotient needed above.

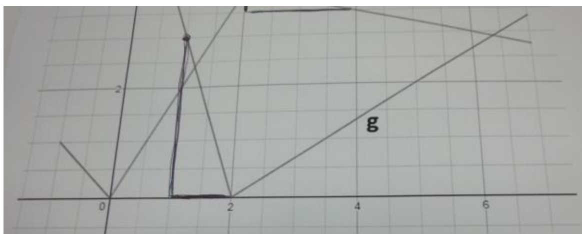


If $u(x) = f(g(x))$, $v(x) = g(f(x))$, and and $w(x) = g(g(x))$, find the following, should they exist:

(a) $u'(1)$

(b) $v'(1)$

(c) $w'(1)$



$u'(1) = f'(g(1)) \cdot g'(1)$ CHAIN

$= f'(3) \cdot g'(1)$

$= \dfrac{-1/2}{2} \cdot \dfrac{-3}{1} = \boxed{\dfrac{3}{4}}$

(B) $v'(1) = g'(f(1)) \cdot f'(1)$

$= g'(2) \cdot f'(1)$

DNE

Does not exist

(C) $w'(1) = g'(g(1)) \cdot g'(1)$

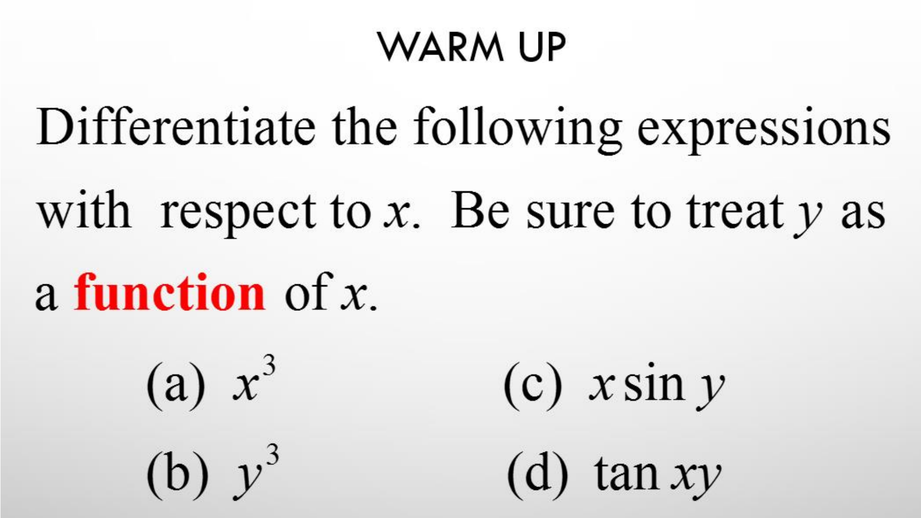
$= g'(3) \cdot g'(1)$

$\dfrac{+1}{3/2}$ $\qquad \dfrac{-3}{1}$

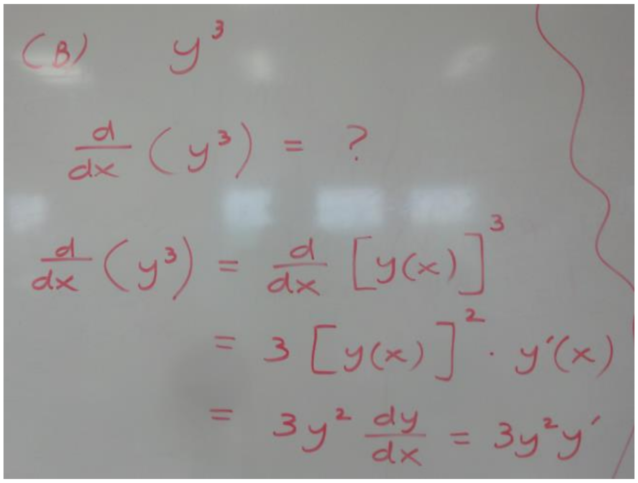
$m = \left(\dfrac{2}{3}\right)(-3)$

$= \boxed{-2}$

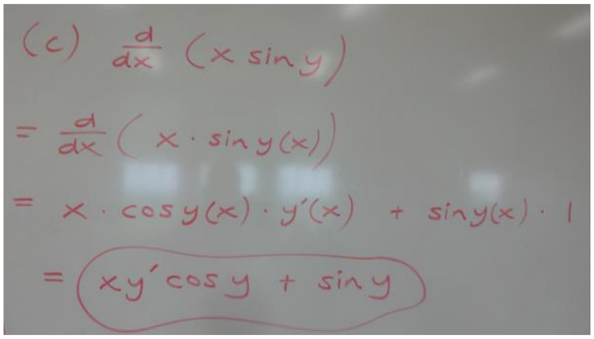
2.6: Implicit Differentiation

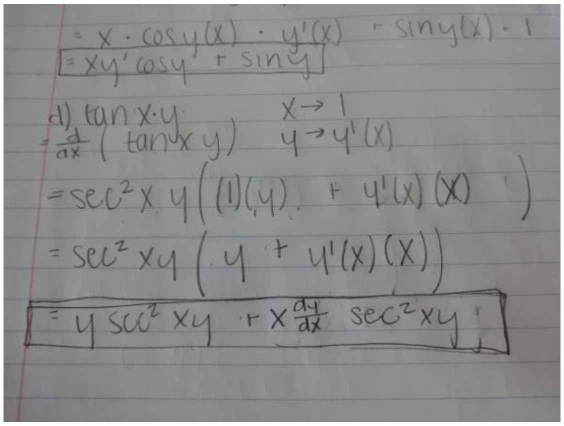


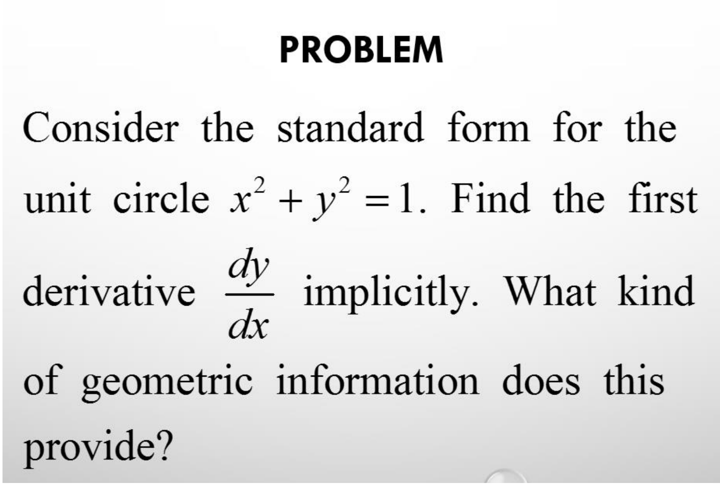
(a) $\frac{d}{dx}\left(x^3\right) = 3x^2$

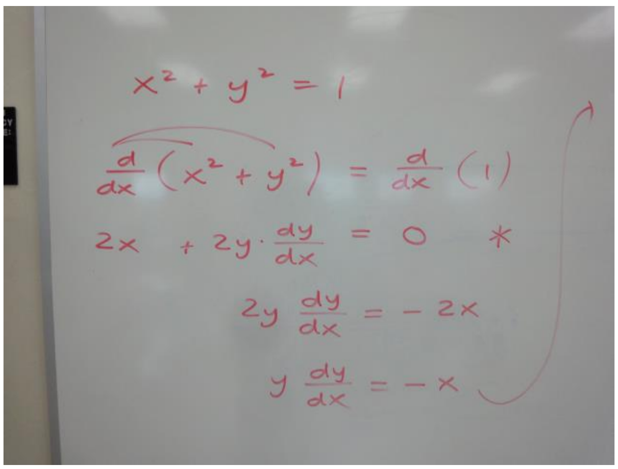


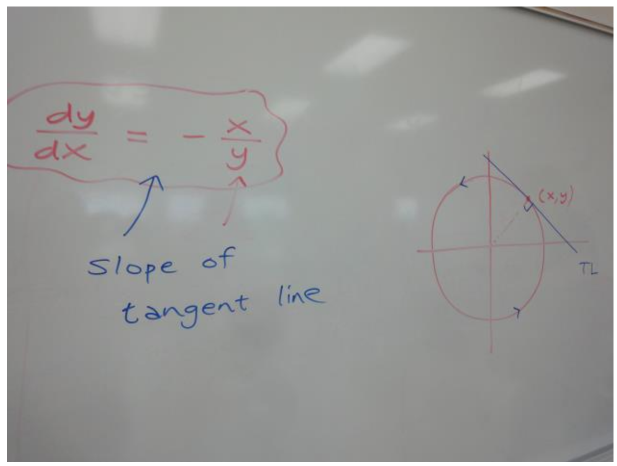
The big connection between (a) and (b) is that *the rule is the same* (Power Rule) but *y* is a function of *x* so we use the Chain Rule in part (b).

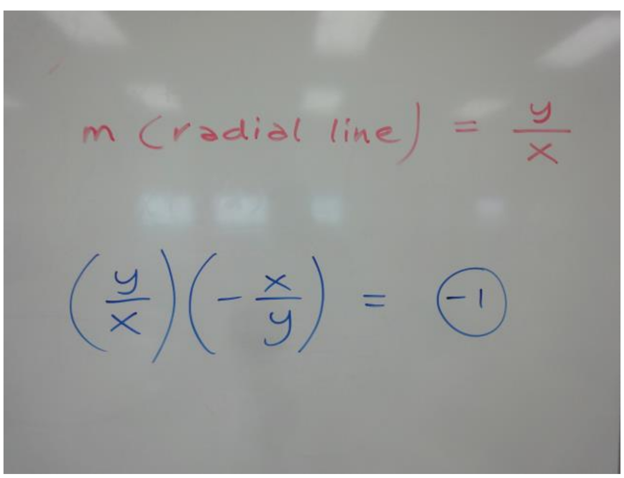












This problem illustrates that the slope of the tangent line at any point (x, y) on the circle is **perpendicular** to the radial line passing through (x, y). This is something we can see in the diagram.