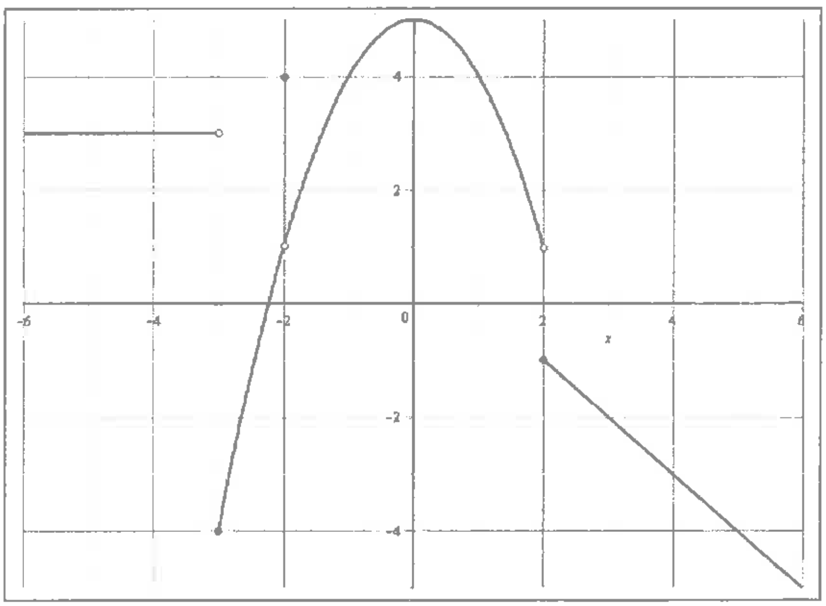
MATH 166
Quiz 1
Fall 2016

Name: Key

DIRECTIONS: This is a <u>closed book</u>, <u>closed notes</u> exam. Calculators are permitted but answers based solely on calculator results are unacceptable. You must still <u>show all work</u> to receive full credit. Good luck.

For problems 1-10, consider the function $y = g(x)$ graphed below. Notice that the graph is shown on the x-window $[-6,6]$ and the y-window $[-5,5]$. Fill in the blanks below based on the diagram or write DNE for "does not exist." Each problem is worth **2 points.**



1. $\lim_{x\to 2^+} g(x) =$ -1

2. $\lim_{x\to 2^-} g(x) =$ 1

3. $\lim_{x\to 2} g(x) =$ DNE

4. $g(2) =$ -1

5. $\lim_{x\to -2^+} g(x) =$ 1

6. $\lim_{x\to -2^-} g(x) =$ 1

7. $\lim_{x\to -2} g(x) =$ 1

8. $g(-2) =$ 4

9. $\lim_{x\to -3^-} g(x) =$ 3

10. True <u>True/False</u>: The graph of $y = g(x)$ is discontinuous on $[-6,6]$.

For exercises 11-14, evaluate the limit analytically. In other words, use some algebra or a well-known Calculus result to arrive at your answer.

11. **(5 points)** $\lim_{x \to -2} \frac{2x^3+1}{x-2}$ $= \frac{2(-2)^3+1}{-2-2} = \frac{-16+1}{-4} = \frac{15}{4}$

(direct substitution)

12. **(5 points)** $\lim_{h \to 2} \frac{h^2-6h+8}{h^2-4}$ $= \lim_{h \to 2} \frac{\cancel{(h-2)}(h-4)}{\cancel{(h-2)}(h+2)}$

$\frac{0}{0}$ Form

$= \lim_{h \to 2} \frac{h-4}{h+2}$

$= \frac{-2}{4} = -\frac{1}{2}$

13. **(5 points)** $\lim_{r \to 4} \frac{r^2-16}{2-\sqrt{r}} \cdot \frac{2+\sqrt{r}}{2+\sqrt{r}}$ $= \lim_{r \to 4} \frac{(r+4)\cancel{(r-4)}^{-1}(2+\sqrt{r})}{\cancel{4-r}}$

$\frac{0}{0}$ Form

$= \lim_{r \to 4} (-1)(r+4)(2+\sqrt{r})$

Note: $r-4 = -1(4-r)$

$= (-1)(8)(4) = -32$

14. **(5 points)** $\lim_{t \to 5} \frac{1/t - 1/5}{5-t}$ $= \lim_{t \to 5} \frac{\frac{5}{5t} - \frac{t}{5t}}{5-t}$

$\frac{0}{0}$ Form

$= \lim_{t \to 5} \left(\frac{1}{\cancel{5-t}} \cdot \frac{\cancel{5-t}}{5t} \right)$

$= \lim_{t \to 5} \frac{1}{5t}$

$= \frac{1}{25}$

$$f(x) = \frac{(x+5)(x-3)}{2(x+3)(x-3)}$$

15. For everything on this page, use the function $f(x) = \dfrac{x^2+2x-15}{2x^2-18}$.

(a) **(6 points)** Determine $\lim_{x\to -3} f(x)$, if possible. Is the function $y = f(x)$ continuous or discontinuous at $x=-3$? If discontinuous, describe the nature of the discontinuity.

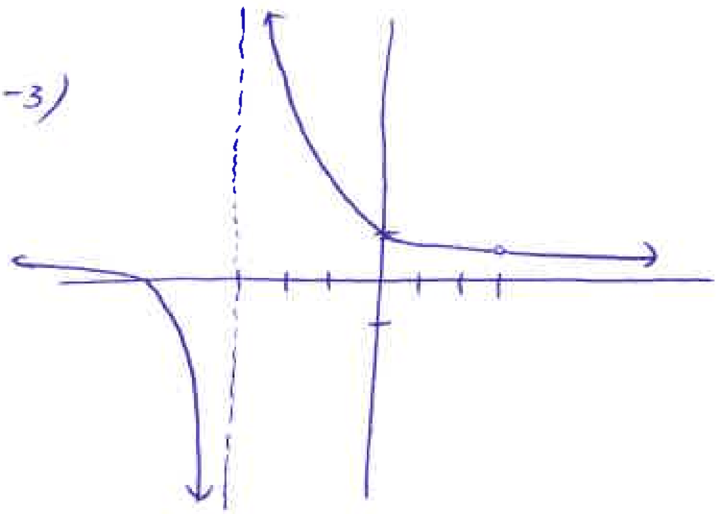
$\lim_{x\to -3} f(x)$ DNE (unbounded)

VA at $x = -3$

(discontinuous at $x=-3$)

As $x \to -3^-$, $f(x) \to -\infty$

As $x \to -3^+$, $f(x) \to +\infty$



(b) **(6 points)** Determine $\lim_{x\to 3} f(x)$, if possible. Is the function $y = f(x)$ continuous or discontinuous at $x=3$? If discontinuous, describe the nature of the discontinuity.

$$\lim_{x\to 3} \frac{(x+5)\cancel{(x-3)}}{2(x+3)\cancel{(x-3)}}$$

$$= \lim_{x\to 3} \frac{x+5}{2(x+3)}$$

$$= \frac{8}{2(6)}$$

$$= \frac{2}{3}$$

discontinuous at $x=3$ (removable discontinuity)

↑

hole at $(3, 2/3)$