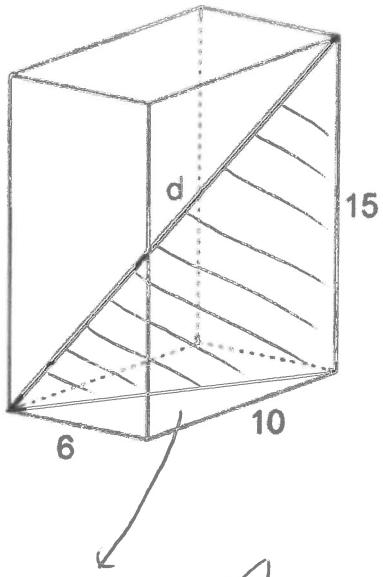
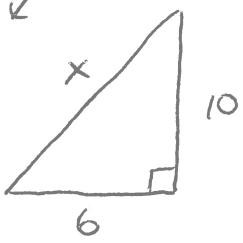
**MATH 126**
**Quiz #2: Pythagorean Theorem; PWW**

**Name:** Key

**Directions:** **Read each question carefully and provide your answers in the space provided. Be sure to answer questions about your reasoning or thinking. You may use a calculator but correct answers without the supporting work will not receive full credit.**

1. **(15 points)** Determine the value of *d* as seen in the diagram. Be sure to show all relevant work and use proper notation.



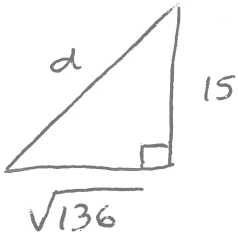




$x^2 = 10^2 + 6^2$

$x^2 = 100 + 36$

$x = \sqrt{136}$

Then we have the shaded triangle above




so $d^2 = 15^2 + (\sqrt{136})^2$

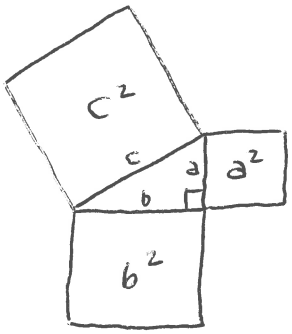
$d^2 = 225 + 136$

$d^2 = 361$

$d = \sqrt{361} = 19$

$\boxed{d = 19}$

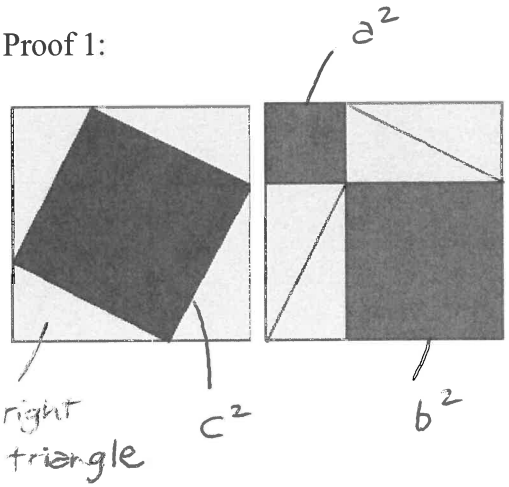
2. (a) **(6 points)** State the Pythagorean Theorem in the clearest terms possible. Explain (in words) what the theorem means to someone who has never seen the theorem.




Given any right triangle w/ legs of length $a$ & $b$ and hypotenuse length $c$, $c^2 = a^2 + b^2$. In other words, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on its legs.

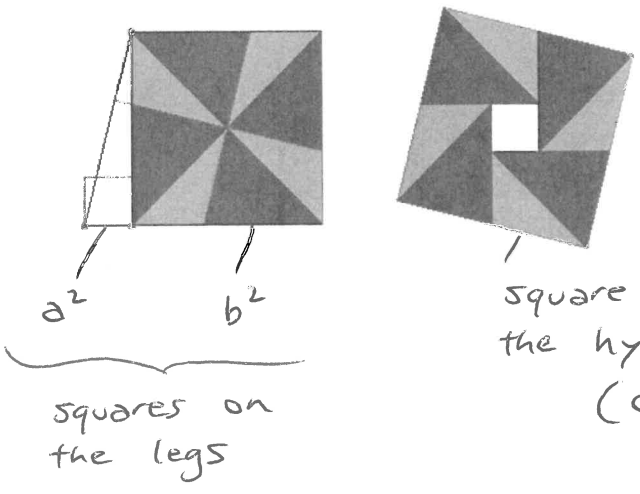
(b) **(12 points)** Below are two "proofs without words" of the Pythagorean Theorem. Explain (in words) how the picture illustrates the Pythagorean Theorem.

Proof 1:




Explanation: The two large squares (side by side) have equal areas & each contains 4 copies of the right triangle. Thus, the dark shading inside both squares must be equal. Therefore, $c^2 = a^2 + b^2$ (see labeling at left).

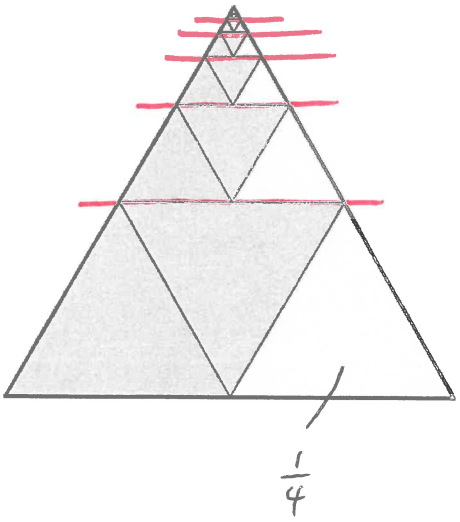
____ Proof 2:




Explanation: The square on the hypotenuse contains all 8 of the triangles that make up $b^2$ and also the small white square ($a^2$). Therefore, $c^2 = a^2 + b^2$.

3. (a) **(8 points)** Below is a proof without words. Explain how the picture provides evidence of the mathematical statement. Be clear in your explanation.

Picture:



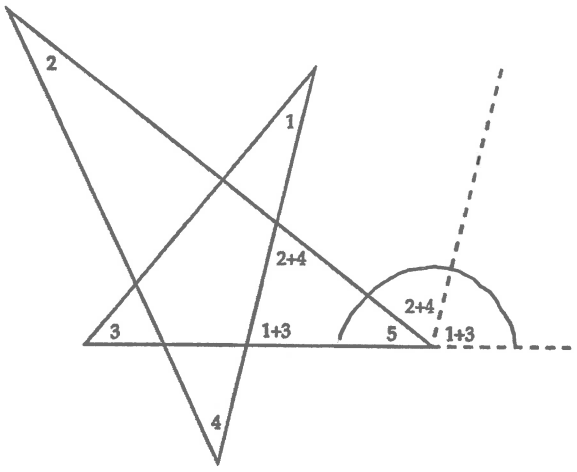

Mathematical statement:

$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\cdots=\frac{1}{3}$$

Focus on one type of triangle (e.g., the lightly shaded ones). The largest light triangle takes up $\frac{1}{4}$ of the entire (large) triangle. The next largest takes up $\frac{1}{16}$ of the entire triangle; the next $\frac{1}{64}$, etc. However, looking across each row of three triangles (see the horizontal lines added on the diagram), the light triangles account for $\frac{1}{3}$ of the space. Thus, $\frac{1}{3} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$.

(b) **(8 points)** Below is a proof without words. What is the mathematical statement that can be inferred from the diagram?

Picture:




Mathematical statement:

The marked angle shows that adding the measures of angles 1, 2, 3, 4, and 5 gives 180°. Looking at the original locations of these angles, we see that adding the interior angles of a "star" gives us 180°.