MATH 166
Quiz 2
Fall 2016

Name: Key

DIRECTIONS: This is a closed book, closed notes exam. Calculators are permitted but answers based solely on calculator results are unacceptable. You must still show all work to receive full credit. Good luck.

1. (a) **(8 points)** Use the **limit definition** of the derivative $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(x)$ for $f(x)=x-3x^2$. Using a shortcut rule doesn't count but you may *check* your answer via a shortcut.

$$f'(x) = \lim_{h\to 0}\frac{x+h-3(x+h)^2-(x-3x^2)}{h}$$

$$= \lim_{h\to 0}\frac{x+h-3x^2-6xh-3h^2-x+3x^2}{h}$$

$$= \lim_{h\to 0}\frac{h(1-6x-3h)}{h}$$

$$= \lim_{h\to 0}(1-6x-3h) = 1-6x$$

$$f'(x) = 1-6x$$

(b) **(5 points)** Find the equation of the tangent line to the graph of $y=f(x)$ at the point $(1,-2)$.

$$f'(1) = 1-6(1)$$

$$= -5$$

$\uparrow$ slope

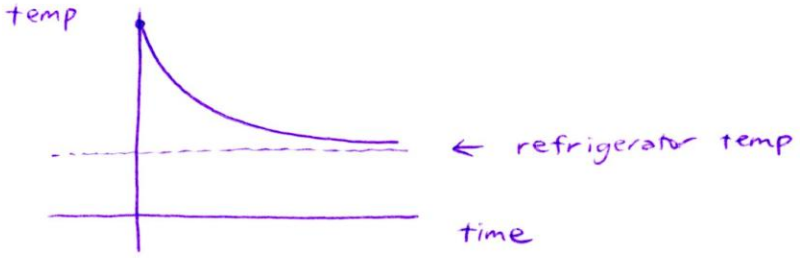
$$y = mx+b$$

$$-2 = (-5)(1)+b$$

$$b = 3$$

$$y = -5x+3$$

2. **(6 points)** A warm can of soda is placed in a cold refrigerator. Sketch (roughly) the graph of the temperature of the soda as a function of time. Is the initial rate of change of temperature greater or less than the rate of change after an hour?

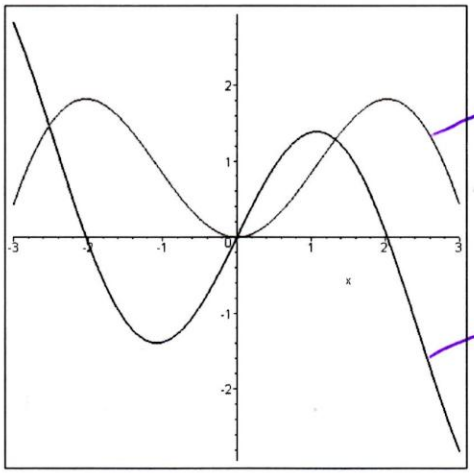


initial ROC is greater at the start (tangent is steeper in the negative sense)

* Most of the cooling happens early on *

3. **(6 points)** Below are the graphs of a function f and its derivative f', both in the same viewing window. Which is which? (Please label them as you see fit.) Write a short summary and support your claims with detailed reasoning.

* The graph of f' crosses the x-axis precisely where f has zero slope.



slopes of f from left to right : +, −, +, −

values of f' start +, go −, go back +, return −

Also see where $f'(x) = 0$ (f has a horizontal TL)

For problems 4-7, find the derivatives. Simplify to a reasonable point (e.g., eliminate complex fractions, negative exponents, etc.)

4. **(6 points)** $f(\theta) = \theta^5 - 7\theta + 9$

$f'(\theta) = 5\theta^4 - 7$

5. **(6 points)** $f(x) = \dfrac{x}{x+1}$

$f'(x) = \dfrac{(x+1)(1) - x(1)}{(x+1)^2}$

$= \dfrac{1}{(x+1)^2}$

(Quotient)

6. **(6 points)** $g(x) = x^2 \sec x$

$g'(x) = x^2 \sec x \tan x + \sec x (2x)$

$= x \sec x (x \tan x + 2)$

(Product)

7. **(6 points)** $h(x) = \cos^4(7x^3)$

$= [\cos(7x^3)]^4$

$h'(x) = 4[\cos(7x^3)]^3 (-\sin(7x^3)) \cdot 21x^2$

Power trig $(7x^3)'$

$h'(x) = -84x^2 \cos^3(7x^3) \sin(7x^3)$

(Chain)