MATH 126 **Name:** Key
Quiz #3: Counting Techniques

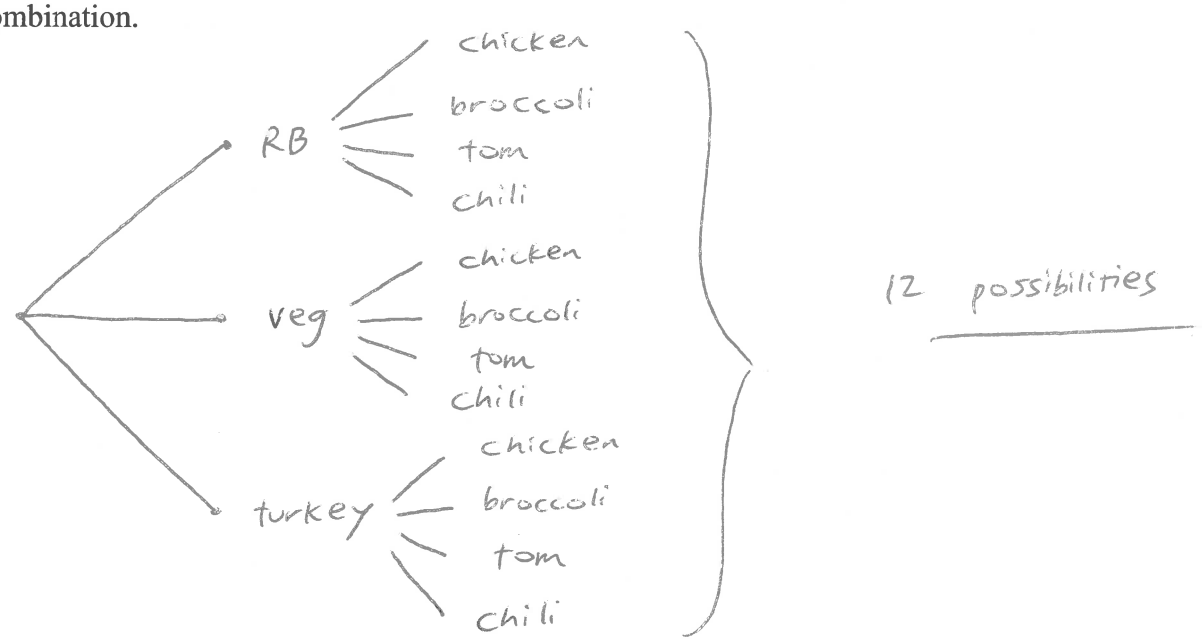
Directions: **Read each question carefully and provide your answers in the space provided. Be sure to show the mathematical structure of your answer (e.g., $6 \cdot 5 \cdot 4 = 120$). You may use a calculator but correct answers without the supporting structure will not receive full credit.**

1. **(4 points)** Explain the difference between a *permutation* and a *combination*. Feel free to give examples if this supports your argument.

A permutation is a particular order / arrangement of objects (the order is significant). For example, ABC and BAC are different permutations of the letters A, B, C. A combination is a selection of objects without regard to order (so there is no distinction between ABC and BAC).

2. You want to order a sandwich and a soup. From the sandwiches, you can choose from roast beef, Mediterranean grilled veggie, or a turkey/bacon club. From the soups, you can choose from chicken noodle, broccoli cheese, tomato, or chili.

(a) **(5 points)** Draw a tree diagram showing all possibilities of a soup and sandwich combination.



(b) **(3 points)** Use a counting technique to find the number of possibilities in (a).

$3 \times 4 = 12$

sandw. soup

3. Standard RI license plates look like this:

(a) **(5 points)** If the style above is followed, how many unique plates are possible?

$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 =$ 676,000

(b) **(5 points)** What if no letters are repeated?

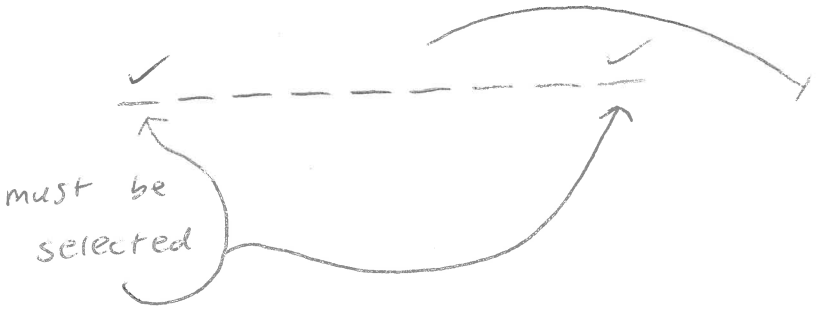
$26 \cdot 25 \cdot 10 \cdot 10 \cdot 10 =$ 650,000

(c) **(5 points)** How about no letters nor numbers repeated?

$26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 =$ 468,000

4. **(8 points)** In how many ways can a student select five essay questions from an exam consisting of nine essay questions? Follow up: How many ways are there if s/he must answer the first question and the last question?

order not relevant: ${}_9C_5 =$ 126 ways



must still pick 3 from the 7:

${}_7C_3 =$ 35 ways

5. **(8 points)** Don't solve these problems—simply identify PERMUTATION (P) or COMBINATION (C). Which counting technique applies? Circle the appropriate letter.

(a) Ten fans at a concert are chosen to go backstage. P (C)
(b) A state elects a governor and lieutenant governor from a pool of 8 candidates. (P) C
(c) A 4-digit PIN is chosen from the numbers 0-9. (P) C
(d) A gardener picks 4 vegetable plants from 11 possibilities for his garden. P (C)

6. **(8 points)** Fifty-two members of a labor union choose a President, VP, Secretary, and Treasurer. In how many different ways can this be done? Note: No member can hold two offices.

$\underset{P}{52} \cdot \underset{VP}{51} \cdot \underset{S}{50} \cdot \underset{T}{49} = 6,497,400$

7. **(10 points)** Do **one** of the two problems below. Place a checkmark in the space next to the problem you are submitting.

_____ Problem A. A four-person crew for the international space station is to be chosen from a candidate pool of 10 Americans and 12 Russians. How many different crews are possible if there must be at least two Russians?

_____ Problem B. From a group of 5 women and 7 men, how many different committees consisting of 2 women and 3 men can be formed? Follow up: What if two of the men are feuding and refuse to serve together on the committee?

Problem A

10 Americans, 12 Russians → 4 person crew

2 Russ: $_{12}C_2 \cdot {}_{10}C_2 = 2970$

3 Russ: $_{12}C_3 \cdot {}_{10}C_1 = 2200$

4 Russ: $_{12}C_4 \cdot {}_{10}C_0 = 495$

$2970 + 2200 + 495 =$ 5,665 possible crews

Problem B

${}_5C_2 \cdot {}_7C_3 = 350$ committees

Follow up:

(feuding men)

7 < 2 feuding, 5 okay

${}_2C_0 \cdot {}_5C_3 = 10$ (contains neither feuding man)

${}_2C_1 \cdot {}_5C_2 = 20$ (contains one feuding man)

$10 + 20 = 30$ groups not containing both feuding men

FTOC: $\underset{\text{men}}{30} \times \underset{\text{women}}{{}_5C_2} = 300$