MATH 166 **Name:** Key
Quiz 3
Fall 2016

DIRECTIONS: This is a <u>closed book</u>, <u>closed notes</u> exam. Calculators are permitted but answers based solely on calculator results are unacceptable. You must still <u>show all work</u> to receive full credit. Good luck.

1. Consider the relation $x^3 + y^3 = 6xy$.

a. **(8 points)** Find $\frac{dy}{dx}$ for the relation above.

$$x^3 + y^3 = 6xy$$
$$3x^2 + 3y^2y' = 6(xy' + y\cdot 1)$$
$$3x^2 + 3y^2y' = 6xy' + 6y$$
$$3x^2 - 6y = 6xy' - 3y^2y'$$
$$3x^2 - 6y = y'(6x - 3y^2)$$
$$y' = \frac{3x^2 - 6y}{6x - 3y^2}$$
$$y' = \frac{x^2 - 2y}{2x - y^2}$$

b. **(4 points)** Find the equation of the tangent line at the point $(3,3)$.

$$y'\Big|_{(3,3)} = \frac{3^2 - 2(3)}{2(3) - 3^2} = \frac{3}{-3} = -1 \leftarrow \text{slope}$$

$$y - 3 = -1(x - 3) \Rightarrow y = -x + 6$$

2. A machine is causing a particle to move along the x-axis so that its position at time t is given by $x(t) = (t-4)^2$, where t is in seconds.

(a) **(4 points)** What is the particle's velocity at $t = 2$?

$$v(t) = x'(t) = 2(t-4)\cdot 1$$
$$v(2) = 2(2-4) = -4 \text{ units/sec}$$

(b) **(7 points)** The machine stops suddenly at $t = 3$ releasing the particle. As the particle continues, where will it be 5 seconds after the machine stops? Explain your thinking.

$x(3) = 1$ (position)
$v(3) = -2$ units/sec

particle is located at 1 & will continue to move 2 units/sec to the left for 5 sec (assuming nothing else acts on the particle)

$$x = 1 - 2(5) = \boxed{-9}$$

location after 5 sec

3. **(8 points)** Locate the absolute extrema of the function $y = 3x^{2/3} - 2x$ on the interval $[-1, 1]$.

$$y' = 3\,\tfrac{2}{3}x^{-1/3} - 2$$

$$= \frac{2}{x^{1/3}} - 2\cdot\frac{x^{1/3}}{x^{1/3}}$$

$$= \frac{2(1-x^{1/3})}{x^{1/3}}$$

$x = 1$ makes $y' = 0$

$x = 0$ makes y' undefined

x	y	
1	1	
0	0	← minimum
−1	5	← maximum

4. **(10 points)** Compare the values of dy and Δy given that $y = \frac{1}{3}x^4$, $x = 2$, $\Delta x = dx = 0.05$. After computing these values, give an interpretation of what these values represent.

$$y = \tfrac{1}{3}x^4$$

$$dy = \tfrac{1}{3}(4x^3)\,dx$$

$$= \tfrac{1}{3}(4\cdot 2^3)(0.05)$$

$$= \boxed{0.5\overline{3}}$$

↑ Change in the tangent line from $x = 2$ to $x = 2.05$

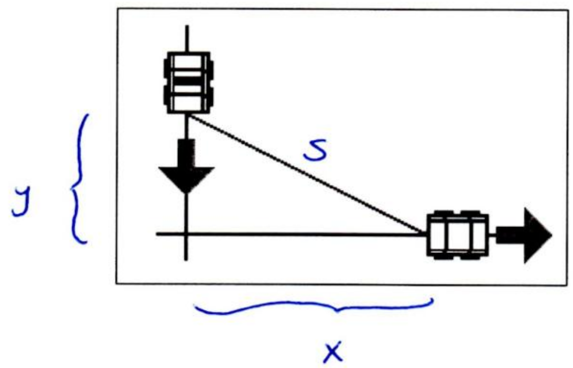
$$\Delta y = y(2.05) - y(2)$$

$$= \tfrac{1}{3}(2.05)^4 - \tfrac{1}{3}(2)^4$$

$$\approx \boxed{0.55367}$$

↑ Change in the function from $x = 2$ to $x = 2.05$

5. **(10 points)** A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 miles north of the intersection and the car is 0.8 miles to the east of the intersection, radar determines that the distance between the two cars is increasing at the rate of 20 mph. If the cruiser is moving at 60 mph at the instant of this reading, what is the speed of the car? See the diagram below.



When $y = 0.6$ and $x = 0.8$,

$\frac{ds}{dt} = 20$ mph

$\frac{dy}{dt} = -60$ mph (y is decreasing),

$\frac{dx}{dt} = ?$

$$s^2 = x^2 + y^2$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$s\frac{ds}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

$$s\frac{ds}{dt} - y\frac{dy}{dt} = x\frac{dx}{dt}$$

When $y = 0.6$ & $x = 0.8$,

$$s = \sqrt{0.8^2 + 0.6^2} = 1$$

$$\frac{1}{x}\left(s\frac{ds}{dt} - y\frac{dy}{dt}\right) = \frac{dx}{dt}$$

$$\frac{1}{0.8}\left(1\cdot(20) - 0.6(-60)\right) = \frac{dx}{dt}$$

$$70 \text{ mph} = \frac{dx}{dt}$$ ← speed of the car