MATH 166
Quiz 4
Fall 2016

Name: Key

DIRECTIONS: This is a closed book, closed notes exam. Calculators are permitted but answers based solely on calculator results are unacceptable. You must still show all work to receive full credit. Good luck.

1. **(20 points)** Give a complete, full analysis of the function

$$f(x)=2x^3-3x^2-12x+12$$

Include intervals of increase/decrease, extrema, concavity, and inflection points (as well as anything else relevant to the graph). Sketch the graph neatly by using all of the information found. **Clearly label your work for full credit.**

$(0,12)$ ← intercept

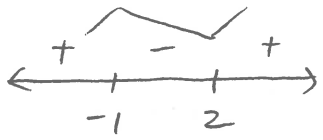
$f(x) = 2x^3 - 3x^2 - 12x + 12$

$f'(x) = 6x^2 - 6x - 12 = 0$ ↑ set

$6(x^2 - x - 2) = 0$

$6(x-2)(x+1) = 0$

$x = -1, 2$ ← critical #s



sign of f'

INC: $(-\infty, -1) \cup (2, \infty)$

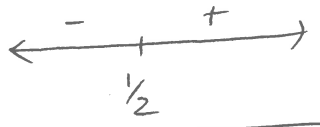
DEC: $(-1, 2)$

Min: $(2, f(2)) = (2, -8)$

Max: $(-1, f(-1)) = (-1, 19)$

$f''(x) = 12x - 6 = 0$ ↙ set

$x = \frac{1}{2}$ ppoi

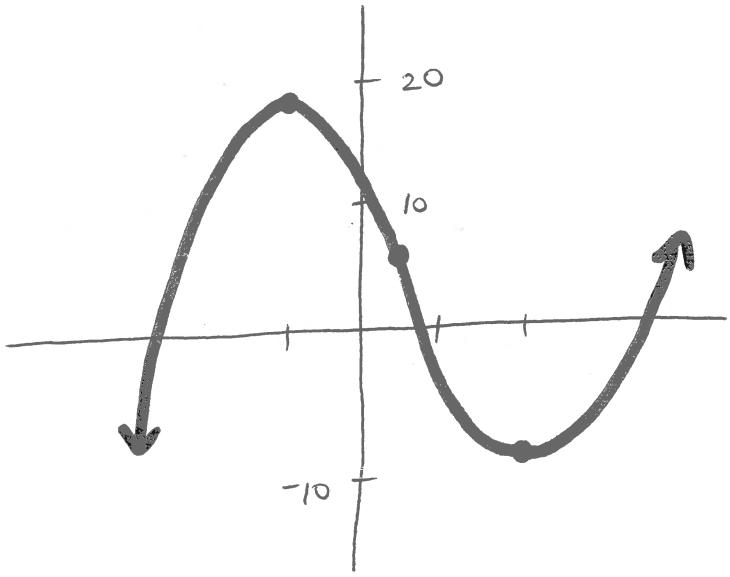


sign of f''

CC↓: $(-\infty, 1/2)$

CC↑: $(1/2, \infty)$

inflection point: $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, \frac{11}{2}\right)$



2. **(10 points)** Where is the function $y = x\sqrt{x+3}$ <u>increasing</u>? Do enough mathematical work to clearly answer this question but you need not supply a graph.

$x + 3 \geq 0$

$x \geq -3$ (Domain)

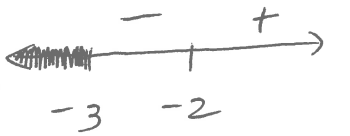
$y' = x \cdot \frac{1}{2}(x+3)^{-1/2} \cdot 1 + \sqrt{x+3} \cdot 1$

$= \frac{x}{2\sqrt{x+3}} + \sqrt{x+3} \cdot \frac{2\sqrt{x+3}}{2\sqrt{x+3}}$

$= \frac{x + 2(x+3)}{2\sqrt{x+3}}$

$= \frac{3x+6}{2\sqrt{x+3}} = 0$ ← set

$x = -2$ ← critical #



sign of y'

Increasing on $(-2, \infty)$

3. **(10 points)** The Jamaican sprinter Usain Bolt set a world record of 9.58 seconds in the 100 meter dash in the summer of 2009. Did his speed ever exceed 37 km/hr during the race? How do you know? Give detailed reasoning.

Average Speed $= \frac{100\text{ m}}{9.58\text{ s}} \approx 10.44\text{ m/s}$

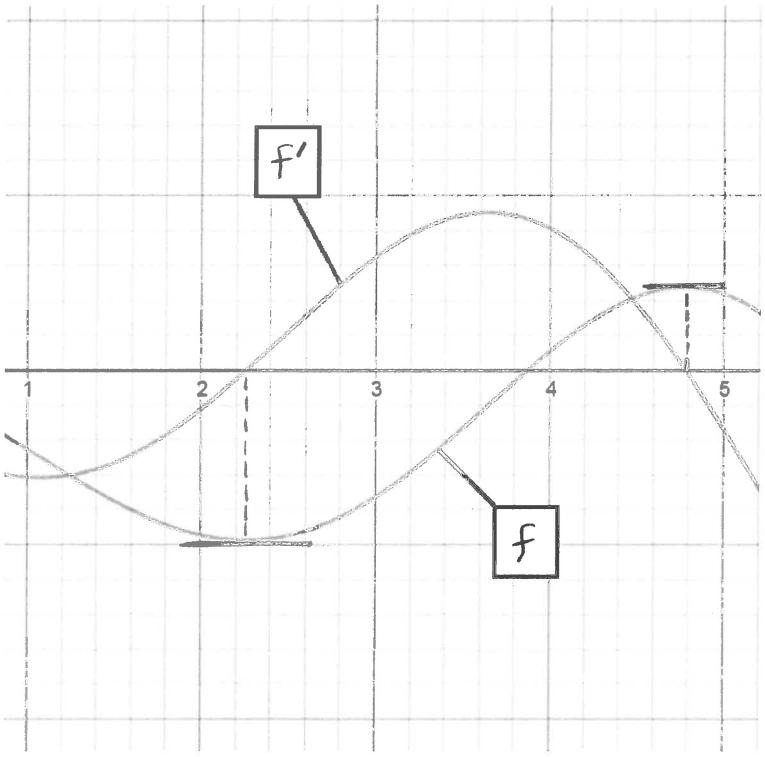
$10.44 \frac{\cancel{m}}{\cancel{s}} \times \frac{3600\cancel{s}}{1\text{ hr}} \times \frac{1\text{ km}}{1000\cancel{m}} \approx$ **37.58 km/hr**

↑ Average speed

The Mean Value Theorem guarantees an instant during the race when Bolt was running <u>exactly</u> 37.58 km/hr (Instantaneous speed).

So the answer is <u>Yes</u>.

4. **(10 points)** Below are sketches of a function and its derivative. Which is which? (Write f or f' in the boxes to make your selection.) Then explain at least two features of the graph(s) that support your decision.



Feature 1: f' is passing through the x-axis ($f' = 0$) where we see horizontal tangents to f (slope = 0). See graph.

Feature 2: For all of $x < 2$, the f graph is decreasing. We see that $f' < 0$ in the corresponding section.