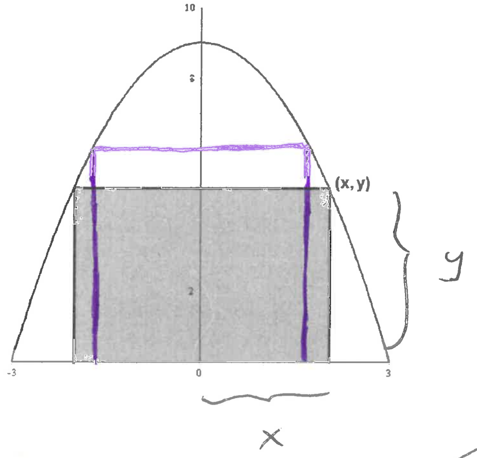
MATH 166
Quiz 5
Fall 2016

Name: Key

DIRECTIONS: This is a closed book, closed notes exam. Calculators are permitted but answers based solely on calculator results are unacceptable. You must still show all work to receive full credit. Good luck.

1. **(12 points)** Find the dimensions of the rectangle of greatest area that has its base on the x-axis and is inscribed in the parabola $y = 9 - x^2$. See the diagram. **Show all work.**



$A = (2x)y$

$= 2xy$

$= 2x(9-x^2)$

$A(x) = 18x - 2x^3$

$0 < x < 3$ for a viable solution

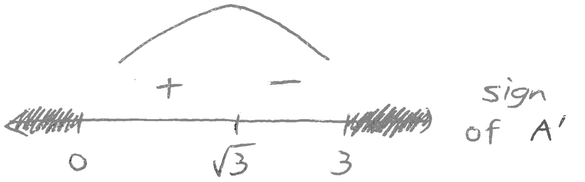
$A'(x) = 18 - 6x^2 \overset{\text{set}}{=} 0$

$6x^2 = 18$

$x^2 = 3$

$x = \pm\sqrt{3}$

Test $x = \sqrt{3}$



$\Rightarrow$ Maximum area at $x = \sqrt{3}$

length $= 2x = 2\sqrt{3}$

width $= y = 9 - x^2$

$= 9 - (\sqrt{3})^2 = 6$

Ans: $2\sqrt{3}$ by 6 rectangle

Area $= (2\sqrt{3})(6)$

$= \boxed{12\sqrt{3} \text{ units}^2}$

(see sketch above)

2. **(10 points)** Use Newton's method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ to approximate the zero of the function $f(x) = x^3 + x - 1$. Use $x_1 = 1$ as an initial guess and find x_2, x_3, and x_4. Show the work/set-up for the computation of x_2; otherwise let the calculator do the work (you can just summarize the work in table form).

$$f(x) = x^3 + x - 1$$

$$f'(x) = 3x^2 + 1$$

$$x_2 = x_1 - \frac{x_1^3 + x_1 - 1}{3x_1^2 + 1}$$

$$= 1 - \frac{1^3 + 1 - 1}{3(1)^2 + 1}$$

$$= 0.75$$

n	x_n
1	1
2	0.75
3	0.686
4	0.682

Actual intercept ≈ 0.6823278

3. **(10 points)** Determine the function $f(x)$ given that $f'(x) = \frac{x^2 + \sqrt{x}}{x}$ and $f(1) = 3$.

$$f'(x) = \frac{x^2}{x} + \frac{\sqrt{x}}{x}$$

$$= x + x^{-1/2}$$

$$f(x) = \frac{x^2}{2} + \frac{x^{1/2}}{1/2} + c$$

$$f(x) = \frac{x^2}{2} + 2\sqrt{x} + c$$

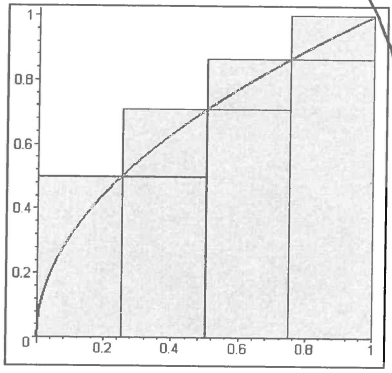
Then $f(1) = \frac{1}{2} + 2\sqrt{1} + c = 3$

$$\Rightarrow \quad c = \frac{1}{2}$$

$$\boxed{f(x) = \frac{x^2}{2} + 2\sqrt{x} + \frac{1}{2}}$$

4. **(8 points)** Use an upper or lower sum (identify your choice below) to approximate the area of the region using the given number of rectangles (of equal width). The function shown is $y = \sqrt{x}$ and the interval is $[0,1]$.

I'm calculating the ________________________ sum. (Fill in the blank with **upper** or **lower** and then show your calculations below.)



Upper

$f(\frac{1}{4})\frac{1}{4} + f(\frac{2}{4})\frac{1}{4} + f(\frac{3}{4})\frac{1}{4} + f(1)\frac{1}{4}$

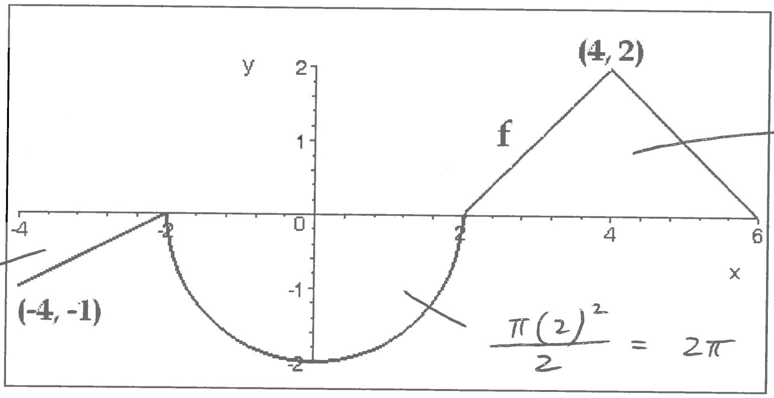
$= \frac{1}{4}\left(\sqrt{\frac{1}{4}} + \sqrt{\frac{2}{4}} + \sqrt{\frac{3}{4}} + \sqrt{1}\right) \approx \boxed{0.768}$

Lower

$f(0)\frac{1}{4} + f(\frac{1}{4})\frac{1}{4} + f(\frac{2}{4})\frac{1}{4} + f(\frac{3}{4})\frac{1}{4}$

$= \frac{1}{4}\left(\sqrt{0} + \sqrt{\frac{1}{4}} + \sqrt{\frac{2}{4}} + \sqrt{\frac{3}{4}}\right) \approx \boxed{0.518}$

5. The graph of f consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.



$\frac{1}{2}(4)(2) = 4$

$\frac{1}{2}(2)(1)$

$\frac{\pi(2)^2}{2} = 2\pi$

(a) **(3 points)** $\int_0^2 f(x)\,dx = \boxed{-\pi}$

(b) **(3 points)** $\int_{-4}^{6} f(x)\,dx = -1 - 2\pi + 4 = \boxed{3 - 2\pi}$

(c) **(3 points)** $\int_{-4}^{6} |f(x)|\,dx = 1 + 2\pi + 4 = \boxed{5 + 2\pi}$