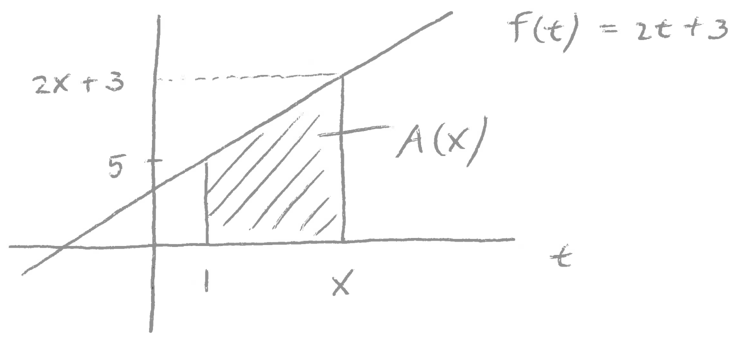
MATH 166
Quiz 6
Fall 2016

Name: Key

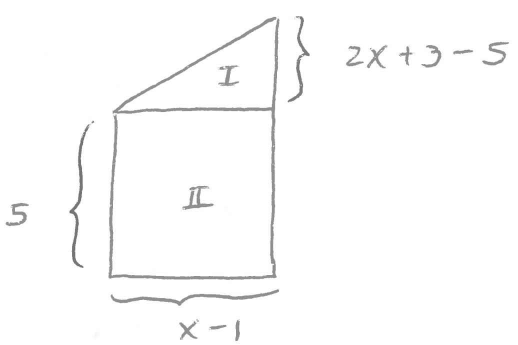
DIRECTIONS: This is a closed book, closed notes exam. Calculators are permitted but answers based solely on calculator results are unacceptable. You must still show all work to receive full credit. Good luck.

1. Consider the area/accumulation function $A(x)=\int_1^x f(t)\,dt$ with $f(t)=2t+3$.

(a) **(4 points)** Draw a picture that shows the area defined by $A(x)$ above.



(b) **(3 points)** Determine an algebraic formula for $A(x)$ by using part (a) above. Note: You shouldn't be using any Calculus.



$$\text{Area}(I) = \tfrac{1}{2}(x-1)(2x-2) = x^2-2x+1$$

$$\text{Area}(II) = 5(x-1)$$

$$A(x) = I + II = x^2-2x+1+5x-5 = \boxed{x^2+3x-4}$$

(c) **(3 points)** Clearly demonstrate the Fundamental Theorem of Calculus (Part I) given the information above. That is, show that $A'(x)=f(x)$.

$$A(x) = x^2+3x-4$$

$$A'(x) = 2x+3 = f(x) \quad ✓$$

For problems 2-6, evaluate the integral. All details should be shown for full credit.

2. **(8 points)** $\int_0^4 (4-t)\sqrt{t}\,dt$

$$= \int_0^4 \left(4t^{1/2} - t^{3/2}\right) dt$$

$$= 4\frac{t^{3/2}}{3/2} - \frac{t^{5/2}}{5/2} \Big|_0^4$$

$$= \frac{8}{3}t^{3/2} - \frac{2}{5}t^{5/2} \Big|_0^4$$

$$= \frac{8}{3}(8) - \frac{2}{5}(32) = \frac{64}{3} - \frac{64}{5} = \frac{128}{15}$$

3. **(8 points)** $\int \frac{1+\sqrt{x}+x}{\sqrt{x}}\,dx$

$$= \int \left(x^{-1/2} + 1 + x^{1/2}\right) dx$$

$$= \frac{x^{1/2}}{1/2} + x + \frac{x^{3/2}}{3/2} + C$$

$$= 2\sqrt{x} + x + \frac{2}{3}x^{3/2} + C$$

4. **(8 points)** $\int \frac{x}{\sqrt{1-4x^2}}\,dx$

let $u = 1-4x^2$

$$\frac{du}{dx} = -8x \Rightarrow \frac{du}{-8} = x\,dx$$

$$= \int \frac{du/-8}{\sqrt{u}}$$

$$= -\frac{1}{8}\int u^{-1/2}\,du$$

$$= -\frac{1}{8}\frac{u^{1/2}}{1/2} + C = -\frac{1}{4}\sqrt{1-4x^2} + C$$

5. **(8 points)** $\displaystyle\int_0^{\pi/6} \frac{\sin\theta}{\cos^2\theta}\,d\theta$ $\displaystyle = \int_0^{\pi/6} (\cos\theta)^{-2}\sin\theta\,d\theta$

$u = \cos\theta$

$du = -\sin\theta\,d\theta$

$$= -\frac{(\cos\theta)^{-1}}{-1}\Bigg|_0^{\pi/6}$$

$$= \frac{1}{\cos\theta}\Bigg|_0^{\pi/6} = \frac{1}{\cos\frac{\pi}{6}} - \frac{1}{\cos 0}$$

$$= \frac{2}{\sqrt{3}} - 1$$

or

$$\int_0^{\pi/6} \frac{1}{\cos\theta}\,\frac{\sin\theta}{\cos\theta}\,d\theta$$

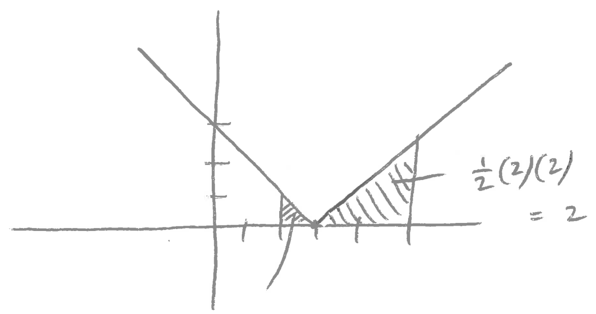
$$= \int_0^{\pi/6} \sec\theta\tan\theta\,d\theta$$

$$= \sec\theta\Big|_0^{\pi/6}$$

$$= \sec\frac{\pi}{6} - \sec 0$$

$$= \frac{2}{\sqrt{3}} - 1$$

6. **(8 points)** $\displaystyle\int_2^5 |x-3|\,dx$



$\frac{1}{2}(2)(2) = 2$

$\frac{1}{2}(1)(1) = \frac{1}{2}$

$$\int_2^5 |x-3|\,dx = \frac{1}{2} + 2$$

$$= 5/2$$