Calculus **Name: ____________________**
Initial Skills Exam (Sample) **Date: _____________________**
Key

Directions: **Answer each question to the best of your ability. Be sure to provide explanations where requested (use the space provided). No calculators please.**

For items 1-3, show all work.

1. TRUE or [FALSE]: For any numbers *a* and *b*, $\sqrt{a^2+b^2}=a+b$. Explain your answer.

   This is false in general. It may work for some choices of *a* and *b*. For example, if $a=5$ and $b=0$ then it is true that $\sqrt{5^2+0^2}=5+0$. However, if $a=2$ and $b=3$ then $\sqrt{2^2+3^2}=2+3$ leads to $\sqrt{13}=5$, which is a false statement.

2. TRUE or [FALSE]: If *x* is any number, then $x^2 \geq x$. Explain your answer.

   This is false in general. Similar to item # 1, this will be true for many choices of *x*. For example, if $x=4$ then it is indeed true that $4^2 \geq 4$. Also, equality holds if $x=0$ or $x=1$. If one chooses a common fractional value such as $x=\frac{1}{2}$, then the statement reads $\left(\frac{1}{2}\right)^2 \geq \frac{1}{2}$ or $\frac{1}{4} \geq \frac{1}{2}$, a false statement.

3. Factor $2x^4-6x^3-8x^2+24x$ completely. Show all work.

   There are numerous ways to approach this. Here is a straightforward method:

$$\begin{aligned} 2x^4-6x^3-8x^2+24x &= 2x\left(x^3-3x^2-4x+12\right) \\ &= 2x\left(x^2(x-3)-4(x-3)\right) \\ &= 2x\left((x-3)\left(x^2-4\right)\right) \\ &= 2x(x-3)(x+2)(x-2). \quad \leftarrow \text{Answer} \end{aligned}$$

Items 4-10 are multiple choice. Circle your answer.

4. Find $\sin\left(\frac{2\pi}{3}\right)$.

   [(a)] $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$

   (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

There are many ways to approach this (recall/memorization, reference angles, special triangles, graph of the sine wave, unit circle, etc.). Using the fact that π radians is equivalent to 180°, $\frac{2\pi}{3}$ radians is equivalent to 120°. Then, recall from the symmetry of the sine wave (or the unit circle) that $\sin 120^{\circ} = \sin 60^{\circ}$. Since $\sin 60^{\circ} = \frac{\sqrt{3}}{2}$, the answer is (a).

5. Find the solution(s) to $\sin^2 \theta = \frac{1}{2}$ on the interval $[0, 2\pi]$.

(a) $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ (b) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

[(c)] $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, and $\frac{7\pi}{4}$ (d) $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, and $\frac{11\pi}{6}$

The statement $\sin^2 \theta = \frac{1}{2}$ implies that $\sin \theta = \pm\sqrt{\frac{1}{2}}$ after taking a square root. Rationalizing the right-hand side reveals the more familiar form $\pm\frac{\sqrt{2}}{2}$ so we have $\sin \theta = \pm\frac{\sqrt{2}}{2}$. Then use your knowledge of key sine values and periodicity to conclude that $\theta = \frac{\pi}{4}$ and $\theta = \pi - \frac{\pi}{4}$ for $\sin \theta = \frac{\sqrt{2}}{2}$ and $\theta = \pi + \frac{\pi}{4}$ and $\theta = 2\pi - \frac{\pi}{4}$ for $\sin \theta = -\frac{\sqrt{2}}{2}$. Thus, the theta values are $\theta = \frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, and $\frac{7\pi}{4}$. The answer is (c).

6. Evaluate $\arctan \frac{\sqrt{3}}{3}$.

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$

[(c)] $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$

The expression $\arctan \frac{\sqrt{3}}{3} = ?$ is equivalent to asking $\tan(?) = \frac{\sqrt{3}}{3}$. You should get 30° or $\frac{\pi}{6}$. The answer is (c).

7. Find the exact solutions to $x^2+4x=7$.

(a) $x=3$ and $x=7$

[(b)] $x=-2+\sqrt{11}$ and $x=-2-\sqrt{11}$

(c) $x=3$ and $x=-\frac{1}{2}$

(d) $x=-2+i\sqrt{3}$ and $x=-2-i\sqrt{3}$

Completing the square or the quadratic formula will lead to the solution. $x^2+4x=7$ leads to $x^2+4x+4=7+4$ or $(x+2)^2=11$ so $x+2=\pm\sqrt{11}$ and finally $x=-2\pm\sqrt{11}$. Choose (b).

8. Given that $f(x)=x^3+5$, find $f^{-1}(x)$.

(a) $f^{-1}(x)=\sqrt[3]{x}-\sqrt[3]{5}$

(b) $f^{-1}(x)=\sqrt[3]{x}-5$

(c) $f^{-1}(x)=\dfrac{1}{x^3+5}$

[(d)] $f^{-1}(x)=\sqrt[3]{x-5}$

Since $f^{-1}(x)$ signifies the inverse function, we should (1) switch *x* and *y*, and then (2) solve for *y*. We are given that $y=x^3+5$ so step (1) gives $x=y^3+5$. Then solving for *y* gives $y^3=x-5$ so $y=\sqrt[3]{x-5}$. Thus, the inverse is given by $f^{-1}(x)=\sqrt[3]{x-5}$, option (d).

9. Write $\ln(x+1)-3\ln(x+2)$ as a single logarithm.

(a) $\dfrac{\ln(x+1)}{\ln(x+2)^3}$

(b) $\dfrac{x+1}{(x+2)^3}$

[(c)] $\ln\dfrac{x+1}{(x+2)^3}$

(d) $\ln\frac{1}{8}$

Using the properties of logarithms,

$$\begin{aligned}\ln(x+1)-3\ln(x+2)&=\ln(x+1)-\ln(x+2)^3\\&=\ln\frac{x+1}{(x+2)^3}.\quad\leftarrow\text{Answer}\end{aligned}$$

The answer is (c).

10. Find the slope of the line passing through the points $(1,-2)$ and $(-3,4)$.

(a) 1

(b) -1

(c) $\dfrac{3}{2}$

[(d)] $-\dfrac{3}{2}$

The slope of the line is given by $m = \frac{\Delta y}{\Delta x} = \frac{4-(-2)}{-3-1} = \frac{6}{-4} = -\frac{3}{2}$. Choice (d) is the correct answer.