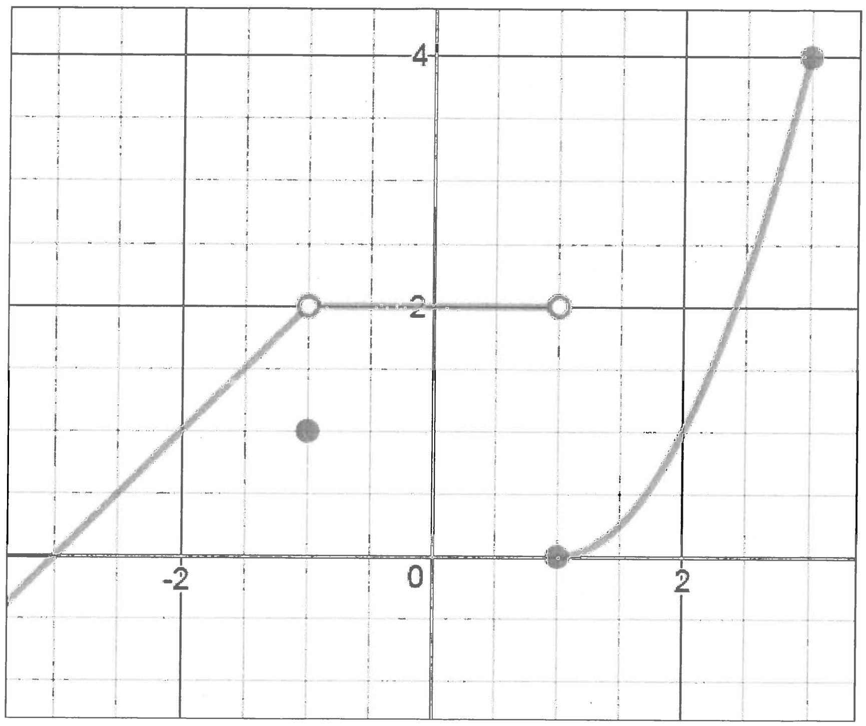
MATH 166
Test 1 / Summer 2017

Name: Key

DIRECTIONS: This is a closed book, closed notes exam. Calculators are permitted but answers based solely on calculator results are unacceptable. You must still show all work to receive full credit. Good luck.

For problems 1-10, consider the function $y = g(x)$ graphed below. Fill in the blanks below based on the diagram or write DNE for "does not exist." Each problem is worth **2 points.** No work needs to be shown.



1. $\lim_{x \to -1^+} g(x) =$ 2

2. $\lim_{x \to -1^-} g(x) =$ 2

3. $\lim_{x \to -1} g(x) =$ 2

4. $g(-1) =$ 1

5. $\lim_{x \to 1^+} g(x) =$ 0

6. $\lim_{x \to 1^-} g(x) =$ 2

7. $\lim_{x \to 1} g(x) =$ DNE

8. $g(1) =$ 0

9. $\lim_{x \to 3^-} g(x) =$ 4

10. False True/False: The graph of $y = g(x)$ is continuous on the window shown.

11. **(6 points)** Use a graph or a table to determine $\lim_{x\to 0}\frac{2\sin(2x)}{5x}$, should it exist. Explain how you obtained your answer.

$f(x) = \frac{2\sin(2x)}{5x}$

look near $x = 0$

x	$f(x)$
-0.03	$.79952$
-0.02	$.79979$
-0.01	$.79995$
0	—
0.01	$.79995$
0.02	$.79979$
0.03	$.79952$

$L = 0.8 = \frac{4}{5}$

(from the table)

For problems 12 and 13, evaluate the limit analytically. In other words, use some algebra or a well-known Calculus result to arrive at your answer.

12. **(5 points)** $\lim_{h\to 2}\frac{h^2-6h+8}{h^2-4}$

$\frac{4-12+8}{4-4} = \frac{0}{0}$

$$= \lim_{h\to 2}\frac{(h-4)\cancel{(h-2)}}{(h+2)\cancel{(h-2)}}$$

$$= \lim_{h\to 2}\frac{h-4}{h+2}$$

$$= \frac{2-4}{2+2}$$

$$= \frac{-2}{4} = -\frac{1}{2}$$

13. **(5 points)** $\lim_{t\to 0}\frac{t}{\sqrt{2t+1}-1} \cdot \frac{\sqrt{2t+1}+1}{\sqrt{2t+1}+1}$

$\frac{0}{0}$

$$= \lim_{t\to 0}\frac{t(\sqrt{2t+1}+1)}{2t+\cancel{1}-\cancel{1}}$$

$$= \lim_{t\to 0}\frac{\cancel{t}(\sqrt{2t+1}+1)}{2\cancel{t}}$$

$$= \lim_{t\to 0}\frac{\sqrt{2t+1}+1}{2}$$

$$= \frac{\sqrt{1}+1}{2} = 1$$

14. **(10 points)** Identify and explain the nature of the discontinuities for the function $f(x) = \dfrac{x^2+2x-8}{x^2-4}$. Are they removable or nonremovable? Support your reasoning with limit statements.

$$f(x) = \frac{(x+4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$$

$$= \frac{x+4}{x+2}, \quad x \neq 2$$

Two discontinuities:

$x = 2, \quad x = -2$

$\underline{x = 2}$ removable discontinuity (limit exists)

$\lim\limits_{x \to 2} f(x) = \frac{6}{4} = \frac{3}{2}.$

$\underline{x = -2}$ nonremovable discontinuity (limit DNE)

As $x \to -2^-$, $f(x) \to -\infty$

As $x \to -2^+$, $f(x) \to +\infty$

VA @ $x = -2$

15. **(5 points)** Find the equation of the tangent line to the graph of $f(x) = x^2 - x$ at the point $(-2, 6)$.

$f'(x) = 2x - 1$

$f'(-2) = 2(-2) - 1$

$= -5$

slope

$y = mx + b$

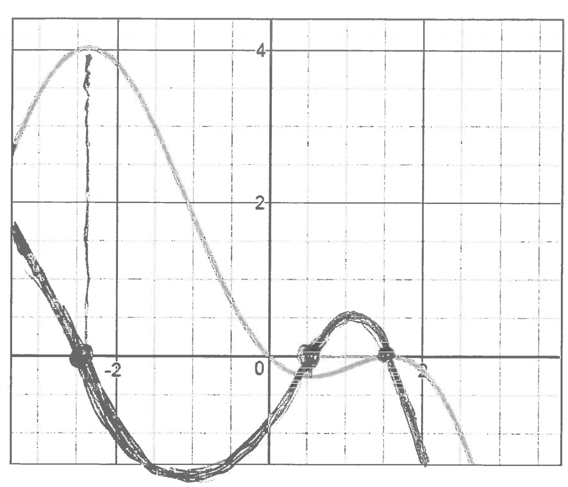
$6 = -5(-2) + b$

$b = -4$

y-intercept

$$\boxed{y = -5x - 4}$$

16. **(5 points)** Given the graph of $y = f(x)$ below, draw the graph of $y = f'(x)$ on the same viewing window.



Rough sketch is fine (need not look exactly like this)

target:
$h'(x) = 1 - 8x$

17. Consider the function $h(x) = x - 4x^2$.

(a) **(6 points)** Use the limit definition to find the derivative.

$$h'(x) = \lim_{h \to 0} \frac{x + h - 4(x+h)^2 - (x - 4x^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{4x^2} - 8hx - 4h^2 - \cancel{x} + \cancel{4x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(1 - 8x - 4h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (1 - 8x - \cancel{4h}) \quad 0$$

$$= 1 - 8x$$

$$\boxed{h'(x) = 1 - 8x}$$

(b) **(3 points)** Calculate $h'(3)$. What does this number mean?

$h'(3) = 1 - 8(3) = -23$ ← slope of the tangent line to $y = h(x)$ at $x = 3$.

18. Find the derivatives. Simplify to a reasonable point.

(a) **(5 points)** $f(x) = (3x+4)(x^2 - 5x)$

$$= 3x^3 - 15x^2 + 4x^2 - 20x$$

$$= 3x^3 - 11x^2 - 20x$$

$$\boxed{f'(x) = 9x^2 - 22x - 20}$$

(product rule possible too)

(b) **(5 points)** $g(x) = \dfrac{4x}{6-x}$

$$g'(x) = \frac{(6-x)4 - 4x(-1)}{(6-x)^2} = \frac{24 - \cancel{4x} + \cancel{4x}}{(6-x)^2}$$

$$= \boxed{\frac{24}{(6-x)^2} = g'(x)}$$