MATH 247
Exam 1
March 3, 2017

Name: Key

Directions: You can use a calculator if you have one but show all work (no cell phones, please). Show all of your work to earn full credit! Good luck!

1. **(10 points)** Consider the problem below:

Mary had 3/4 cup of juice in her glass. She drank 1/3 of it. How much juice is left?

Will the calculation $\frac{3}{4}-\frac{1}{3}$ solve the problem? Why/why not?

No, it will not. In this problem, $\frac{3}{4}$ cup is the whole. If Mary drinks a third of it, then this means she drank $\frac{1}{4}$ cup (not $\frac{1}{3}$ cup). See the diagram:

$\frac{3}{4}$ cup:

2. **(8 points)** Explain why $\frac{2}{3}+\frac{1}{4}\neq\frac{3}{7}$. Be convincing in your explanation.

Explanation #1:

Thirds & Fourths do not give sevenths. The parts of each fraction (thirds vs. fourths) are not equal in size so the addition is not justified.

Explanation #2:

2/3 is fairly close to a whole. Adding 1/4 to it should get us even closer to a whole yet the "answer" 3/7 is less than 1/2. This "method" cannot be valid.

3. **(10 points)** Hector solved the problem $23\frac{2}{5}-17\frac{4}{5}$ and his solution is below.

$$\begin{array}{r} 23\frac{2}{5} = 22\frac{7}{5} \\ -17\frac{4}{5} = 17\frac{4}{5} \\ \hline 5\frac{3}{5} \end{array}$$

Hector couldn't subtract 4/5 from 2/5 so he added 1 (5/5) to 2/5 to make 7/5. In order to compensate for this, Hector changed 23 to 22. Then he subtracted the whole numbers and fractions separately.

Explain what Hector did.

4. **(12 points)** Use 3rd/4th grade reasoning strategies to determine which fraction is larger:

(a) $\frac{1}{6}$ or $\frac{1}{4}$

4 $\frac{1}{4}$s make a whole; 6 $\frac{1}{6}$s make a whole. The $\frac{1}{4}$s require fewer pieces to make the whole so $\frac{1}{4} > \frac{1}{6}$.

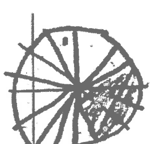
(b) $\frac{3}{7}$ or $\frac{8}{11}$

$3 \times 2 = 6$ so $\frac{3}{6} = \frac{1}{2}$. Since 3 is halfway to 6, 3 is not quite halfway to 7 so $\frac{3}{7} < \frac{1}{2}$. Use similar reasoning w/ 8 & 16 to conclude that $\frac{8}{11} > \frac{1}{2}$. Therefore, $\frac{3}{7} < \frac{1}{2} < \frac{8}{11}$ so $\frac{3}{7} < \frac{8}{11}$.

(c) $\frac{7}{8}$ or $\frac{10}{11}$

Each fraction is just a unit fraction away from the whole. $7/8$ is $1/8$ from 1; $10/11$ is $1/11$ from 1. Use similar reasoning to part (A) to conclude $\frac{1}{8} > \frac{1}{11}$. Thus, $\frac{7}{8}$ is farther from 1 than $\frac{10}{11}$ $\Rightarrow$ $\frac{7}{8} < \frac{10}{11}$.

5. **(10 points)** In the problem below, Taylor was asked to circle the larger fraction. Is Taylor's reasoning valid? Why/why not?

$\frac{4}{9}$ | $\frac{4}{15}$  has more peices

No — Taylor sees the larger number (15) and thinks this fraction is larger. It is common for children to use whole number reasoning when first introduced to fractions.

6. **(15 points)** Suppose you have 5 bottles of Gatorade in your refrigerator. You drink $\frac{3}{4}$ of a bottle every single day. How many days' supply do you have? Notice this problem can be solved by calculating $5 \div \frac{3}{4}$.

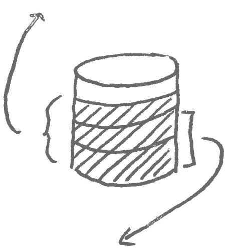
(a) Solve $5 \div \frac{3}{4}$ using any simple method.

$$5 \div \frac{3}{4} = 5 \times \frac{4}{3} = \frac{20}{3} = 6\frac{2}{3} \text{ days supply}$$

(b) Does the word problem describe **partition division** or **measurement division**? How do you know?

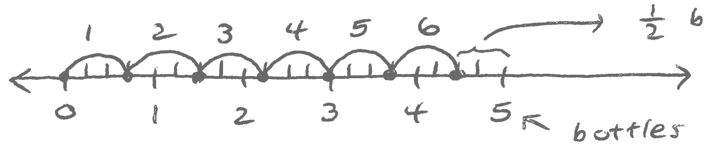
Measurement (this is repeated subtraction of $\frac{3}{4}$ bottle per day)

daily supply



$\frac{1}{2}$ bottle = $\frac{2}{3}$ daily supply

(c) Model the solution to the problem using a number line.



$\frac{1}{2}$ bottle left (the humps are days)

(d) Use the model in (c) to explain the answer in part (a). Reconcile any differences in the remainder from the number line and the remainder from using the standard algorithm.

The model says we have a 6 day supply. There is a ½ bottle remaining. Since a daily supply is 3/4 bottle, a ½ bottle represents 2/3 days supply (see part (A) answer & the picture of the bottle).

(e) Use the model in (c) to justify the "invert and multiply" rule.

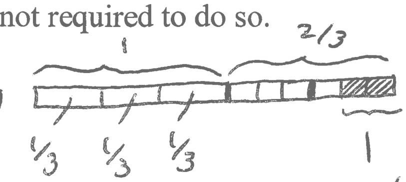
How many $\frac{3}{4}$s are in 1? Ans: $1\frac{1}{3}$ (count the humps in $\underline{1}$ bottle). $1\frac{1}{3} = 4/3$. Then ask: How many $\frac{3}{4}$s fit in 5? Ans: $5 \times \frac{4}{3}$. Thus, $5 \div \frac{3}{4} = 5 \times \frac{4}{3}$.

7. **(9 points)** Virginia goes to Deluxe Candy Land to get a creamy-caramel-delight candy bar. The bar measures exactly $7\frac{2}{3}$ inches. When she gets home, her brother and sister are very jealous. Virginia decides to divide the candy bar equally into three parts. How much of the candy bar does each person get? *Use reasoning (not an algorithm) to solve this problem.* You may want to use a diagram but you are not required to do so.

$7\frac{2}{3}$

2 2 2

$1\frac{2}{3}$ left to share



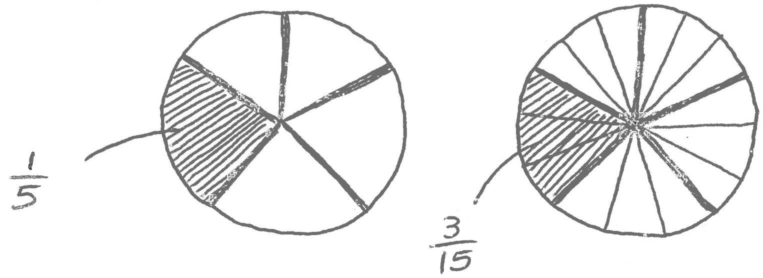
each gets 1/3

each gets 2/9

Ans: $2 + \frac{1}{3} + \frac{2}{9}$

$= 2\frac{5}{9}$ in

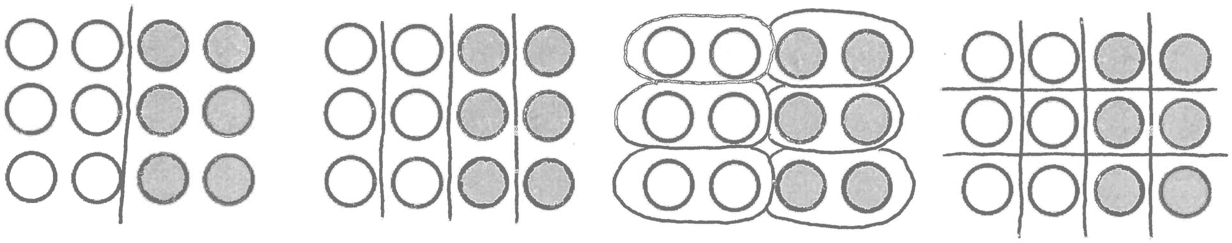
8. **(10 points)** (a) Model the statement $\frac{1}{5} = \frac{3}{15}$ using fraction circles. Make a drawing of this below.



(b) Explain how you know (from the fraction circle model) that the fractions are equivalent.

Equal areas are shaded.

9. **(8 points)** Examine the arrays below. Consider the fraction of chips which are **dark**. Write down a fraction represented by the array and circle/group in a way that is consistent with the fraction. Note: You may not need all of the arrays provided.

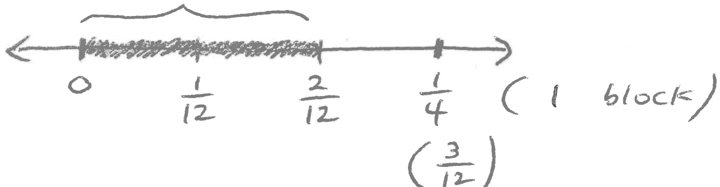


$\frac{1}{2}$ $\qquad$ $\frac{2}{4}$ $\qquad$ $\frac{3}{6}$ $\qquad$ $\frac{6}{12}$

10. **(8 points)** A block is 1/4 mile. John walked 2/3 of a block. How far did he walk? Solve this problem by using

(a) a number line.

$\frac{2}{3}$ of $\frac{1}{4}$ mile



$\frac{2}{12} = \frac{1}{6}$ mile

(b) the standard algorithm.

$\frac{2}{3}$ of block $= \frac{2}{3} \times \frac{1}{4}$ mi $= \frac{2 \times 1}{3 \times 4} = \frac{2}{12} = \frac{1}{6}$ mile