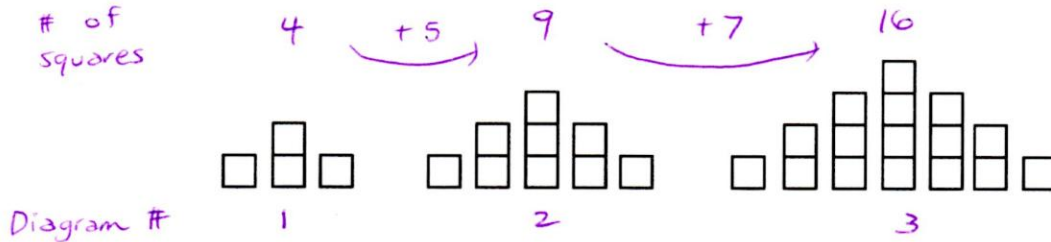MATH 246

Name: _____Key_____

Exam 1

October 7, 2016

Directions: You can use a calculator if you have one but show all work (no cell phones, please). Show all of your work to earn full credit! Good luck!

1. Consider the pattern below (the first three diagrams are shown):

(a) **(5 points)** How many squares will you see in the fifth diagram? How do you know?

Diagram #	1	2	3	4	5
# of squares	4	9	16	25	36

36 squares;
Add 9 to 16 → 25
Add 11 to 25 → 36

(b) **(5 points)** How many squares will you see in the nth diagram? How do you know?

Diagram 5 has 36 squares or $(5+1)^2$ squares.

Diagram n has $(n+1)^2$ squares.

(c) **(7 points)** Will there ever be a diagram with exactly 200 squares? If not, which diagram contains the closest to 200 squares?

Can $(n+1)^2$ ever equal 200? Notice that if $n=13$ (diagram # 13), we get $(13+1)^2 = 14^2 = 196$ squares. The next diagram will far exceed 200 squares so <u>NO</u> is the answer. $n=13$ is closest.
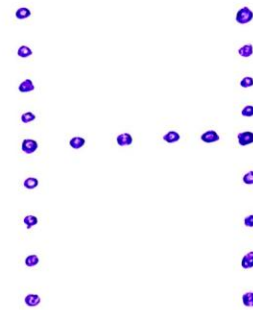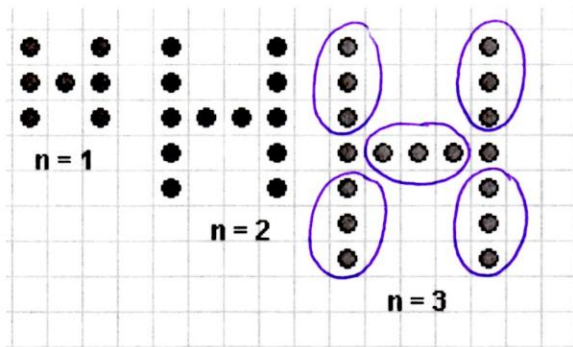
2. **(8 points)** When two pieces of rope are placed end to end, their combined length is 130 feet. When the two pieces are placed side by side, one is 26 feet longer than the other. What are the lengths of the two pieces?
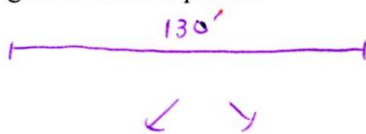
$65 + 13 = \boxed{78 \text{ ft}}$

$65 - 13 = \boxed{52 \text{ ft}}$

3. Consider the H pattern below:

(a) **(4 points)** Draw the next object that corresponds to $n = 4$. You can draw it above. ↑

(b) **(5 points)** Find an algebraic expression that describes the number of dots in diagram n.

n	# dots
1	7
2	12
3	17
4	22

7) +5
12) +5
17) +5
22) +5

n	# dots
0	2

$5n + 2$

(c) **(6 points)** Circle the parts of the H that fit the expression from part (b) and explain (write a sentence) how you know the equation fits the pattern.

(See above) When $n = 3$, we circle

5 groups of $n (= 3)$ with two dots

left over $\Rightarrow$ $\underline{\underline{5n + 2}}$

4. **(8 points)** Someone plays the following game with you: They ask you to pick any number. Add 4 to it. Multiply the answer by 6. Subtract 9. Divide by 3. Add 13. Divide by 2. Subtract your original number. Then they say: **I know your number!** What is the number? How is this possible? Explain.

Pick a number (call it x)

$x + 4$ (Add 4)

$6(x + 4)$ (times 6)

$= 6x + 24$

$6x + 24 - 9 = 6x + 15$

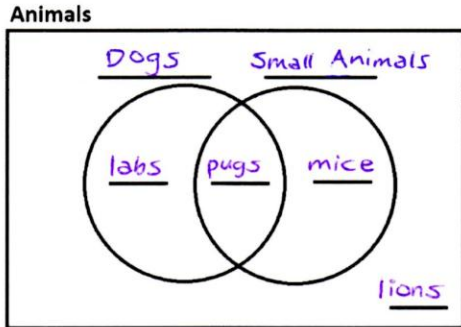$6x + 15 = 3(2x + 5)$

Divide by 3 $\Rightarrow$ $2x + 5$

$2x + 18$ (Add 13)

$2(x + 9)$

Divide by 2: $x + 9$

$x + 9 - x = \boxed{9}$ (Algebra!)

5. **(18 points)** In the universe of "animals," (1) define two sets that make sense and then (2) give an example of things that go into each part of the Venn diagram.

Animals

Dogs   Small Animals

labs   pugs   mice

lions

6. **(8 points)** Kelsey has 70 Pokémon cards total.
She only has 12 Pokémon with more than 100 HP.
All of her Bug type Pokémon have less than 100 HP.
Of her Bug type Pokémon, half have wings and the other half do not.
She has 52 Pokémon that are not bug type. ⟵   18 bug
21 of her Pokémon with less than 100 HP have wings.
25 of her Pokémon have wings.
How many of her Pokémon have wings and more than 100 HP? ⟍

see diagram

$$\underline{\underline{4}} \quad \text{(Answer)}$$

> 100 HP   Bug

0   9

0

4   9

12

12
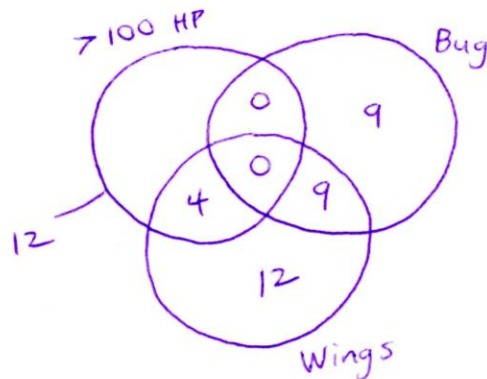
Wings

7. John writes the following statement: $4 \times 5 = 20 + 4 = 24 \div 6 = 4$.

(a) **(4 points)** Explain what is wrong with the mathematical statement above.
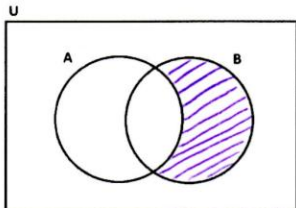
The equals sign does not convey "balance" or "is equal to". Ex: $4 \times 5 \neq 20 + 4$

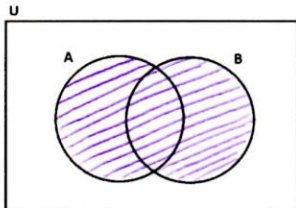(b) **(5 points)** Fix/repair the statement so it reads correctly.

Ex: $(4 \times 5 + 4) \div 6 = 4$

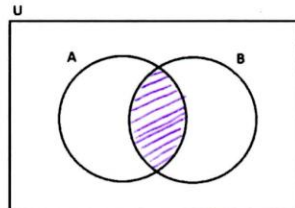8. **(10 points)** For the following problems, sketch the appropriate part of the Venn diagram.
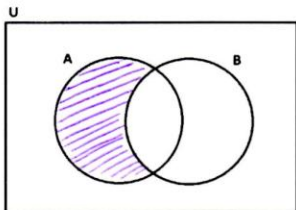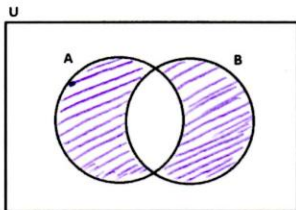
$B - A$

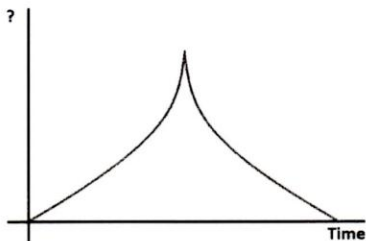$A \cup B$

$A \cap B$

$A \cap \bar{B}$

$A \cup B - A \cap B$

9. **(7 points)** Define "?" so that this graph would be representative of what happens with the passage of time. Be clear in your explanation.

Many examples are possible:

1) intensity level of a severe storm

2) heart rate / pulse during a workout (followed by a cool down period).

etc.