MTH 150
Exam 2
Fall 2014

DIRECTIONS: This is a <u>closed book</u>, <u>closed notes</u> exam. <u>No electronic devices</u> are allowed (this means calculators, computers, cell phones, pagers, etc.). Be neat and <u>show all work</u> to receive full credit. Correct answers without the supporting evidence to back it up receive only partial credit. Good luck.

1. (a) **(6 points)** Based on the **definition of the derivative** find $f'(x)$ for $f(x)=4-x^2$. Using a shortcut rule doesn't count but you may *check* your answer via a shortcut.

$f'(x) = -2x$

$f'(x)=\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f'(x)=\lim_{h\to 0} \frac{4-(x+h)^2-(4-x^2)}{h}$

$f'(x)=\lim_{h\to 0} \frac{4-(x^2+2xh+h^2)-4+x^2}{h}$

$f'(x)=\lim_{h\to 0} \frac{\cancel{4}-\cancel{x^2}-2xh-h^2-\cancel{4}+\cancel{x^2}}{h}$

$f'(x)=\lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}}$ ✓

$f'(x)=\lim_{h\to 0} -2x-\cancel{h}^{\to 0}$

$\boxed{f'(x)=-2x}$ ✓

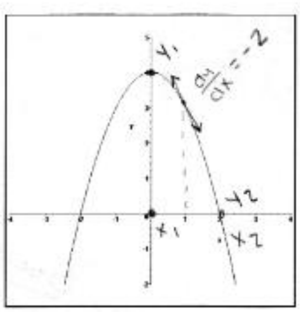
(b) **(3 points)** Find the value of $f'(1)$ by using part (a) above.

$f'(x)=-2x$

$f'(1)=-2(1)$

$\boxed{f'(1)=-2}$ ✓

(c) **(3 points)** What does the number from part (b) mean? Given the graph of $f(x)=4-x^2$ below, give a convincing explanation.



At $x=1$ on the graph, the slope of the tangent line is equal to -2 ✓

- on the graph, the slope is negative at $x=1$, given the graph, you can use the slope equasion to find the slope of the tangent line @ $x=1$

$\frac{y_2-y_1}{x_2-x_1} = \frac{0-4}{2-0} = \frac{-4}{2} = \underline{-2}$

For problems 2-4, find the derivatives. Simplify to a reasonable point (e.g., eliminate complex fractions, negative exponents, etc.)

2. **(7 points)** $y = x^2 \ln x$

$y' = x^{\cancel{2}} \cdot \frac{1}{\cancel{x}} + 2x \ln x$

$= x + 2x \ln x$

3. **(7 points)** $f(x) = \dfrac{1+\sin x}{\cos x}$ $\rightarrow \cos x$, $\rightarrow -\sin x$

$f'(x) = \dfrac{\cos x(\cos x) - [-\sin x(1+\sin x)]}{\cos^2 x}$

$= \dfrac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x}$

$= \dfrac{1 + \sin x}{\cos^2 x}$

4. **(7 points)** $g(\theta) = \tan^4(3\theta)$

$g'(\theta) = 4\tan^3(3\theta)\sec^2(3\theta)(3)$

$= 12\tan^3(3\theta)\sec^2(3\theta)$

Good

5. **(8 points)** Find the derivative $\frac{dy}{dx}$ of the implicitly-defined relation given by $x^2 + \cos y = xe^y$.

$$\frac{dy}{dx}(x^2 + \cos y) = \frac{dy}{dx}(xe^y)$$

$$= 2x + (-\sin y)\left(\frac{dy}{dx}\right) = xe^y \cdot \frac{dy}{dx} + e^y ✓$$

$$= \cancel{2x} - \sin y \cdot \frac{dy}{dx} = \cancel{xe^y \cdot \frac{dy}{dx}} + e^y$$

$$-2x \qquad -xe^y \cdot \frac{dy}{dx}$$

$$x \cdot e^y \quad (uv+vu)$$
$$u \quad v$$
$$= (x)(e^y \cdot y') + (e^y)(1)$$
$$= xe^y \cdot y' + e^y$$

$$-\sin y \cdot \frac{dy}{dx} - xe^y \cdot \frac{dy}{dx} = e^y - 2x$$

$$\frac{\frac{dy}{dx}(-\sin y - xe^y)}{-\sin y - xe^y} = \frac{e^y - 2x}{-\sin y - xe^y} \rightarrow \frac{dy}{dx} = \frac{e^y - 2x}{-\sin y - xe^y}$$

Fine

$$\frac{dy}{dx} = \frac{e^y - 2x}{-(\sin y + xe^y)} ✓ \ ?$$

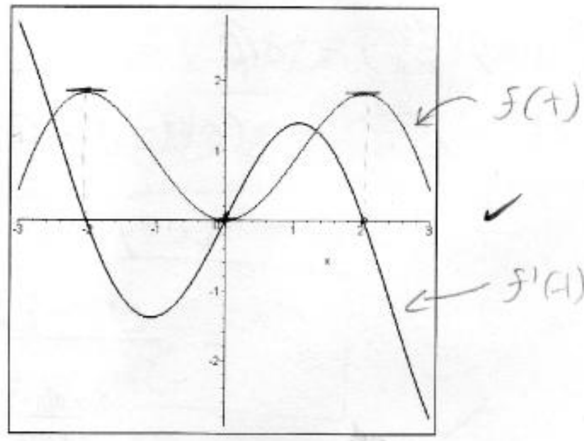
6. **(8 points)** Consider finding the derivative of the function $y = (3x-7)^2$.

John L. says: I'd use the power rule and chain rule.
Kaylee M. says: I'd use the product rule.
Karen D. says: I'd just use the sum/difference rules with the basic functions.

Demonstrate that all of the above are viable options by showing that each suggestion leads to the same result.

$y = (3x-7)^2$	$y = (3x-7)^2$	$y = (3x-7)^2$
$y = u^2 \quad y' = 2u$	$(3x-7)(3x-7)$ (u, v)	$(3x-7)(3x-7)$
$u = 3x-7 \quad u' = 3$	$uv + vu$	$= 9x^2 - 21x - 21x + 49$
$2(3x-7) \cdot 3$	$(3x-7)(3) + (3x-7)(3)$ ✓	$9x^2 - 42x + \cancel{49}$ ✓
$6(3x-7)$ ✓	$9x - 21 + 9x - 21$	$2 \cdot 9x - 42$
$y' = 18x - 42$	$y' = 18x - 42$	$y' = 18x - 42$

7. **(6 points)** Below are the graphs of a function f and its derivative f', both in the same viewing window. Which is which? (Please label them as you see fit.) Write a short summary and support your claims with detailed reasoning.



where ever f(t) has tangent lines of a zero slope, f'(t) is going to have a value of zero, also, when wherever f(t) has a + slope, f(t) will have positive values, the same holds true for when f(t) has a − slope, f'(t) will have − values. ∴ the labels on the graph correspond to each graph.

Good

8. **(6 points)** Find the equation of the line tangent to the graph of $f(x) = xe^x$ at the point $x = 0$.

$f'(t) = (t \cdot e^t \cdot 1) + e^t \cdot 1$

$f'(x) = xe^x + e^x$

$f'(0) = 0 \cdot 1 + 1 = 1$

$f(0) = 0 \cdot 1 = 0$

$0 = 1(0) + b$

$b = 0$

$\boxed{y = x}$

9. **(6 points)** Express, in the simplest way possible, the tenth derivative of $y = e^{2x}$.

$y^{(1)} = e^{2t} \cdot 2 = 2e^{2t} = 2^1 e^{2t}$

$y^{(11)} = 2e^{2t} \cdot 2 = 4e^{2t} = 2^2 e^{2t}$

$y^{(111)} = 4e^{2t} \cdot 2 = 8e^{2t} = 2^3 e^{2t}$

$\boxed{y^{(10)} = 2^{10} e^{2t}}$ $\boxed{y^{(10)} = 1024 e^{2t}}$

4 = 16
4 = 16 > 64
2 = 4
4·10 = 640
4·5 = 320
64·1 = 64
1024

-0

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DIRECTIONS: THERE ARE FOUR PROBLEMS ON THIS PART OF THE TEST; DO ANY THREE. Clearly indicate the one you are skipping. Calculators are permitted. However, answers based solely on calculator results are unacceptable. You must still show all work to receive full credit. Good luck.

10. In a fireworks display, a shell is launched vertically upward from the ground, reaching a height (in feet) of $s(t) = -16t^2 + 256t$ after t seconds. The shell is designed to burst when it reaches maximum altitude.

(a) **(4 points)** When will the shell burst? $s(t) = -16t^2 + 256t$

$s'(t) = -16 \cdot 2t + 256 = -32t + 256$

$-32t + 256 = 0$
$-256 \quad -256$
$\frac{-32t}{-32} = \frac{-256}{-32}$

$\boxed{t = 8 \text{ seconds}}$ ✓

(b) **(4 points)** Why is the derivative important to part (a)? The derivative in part 'A' defines the velocity of the firework. ✓ We are told that the firework explodes at maximum altitude. By setting the derivative equal to zero, we can tell how long the rocket takes to reach its apex, thus determining flight time before explosion. ✓

(c) **(3 points)** What is the altitude of the shell the instant it explodes?

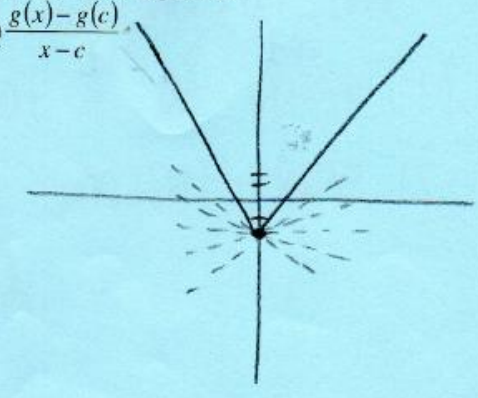
$s(8) = -16(64) + 256(8) = \boxed{1024 \text{ ft}}$ ✓

11. **(11 points)** Explain why the function $g(x) = |x+2|$ is not differentiable at the point $c = -2$. You should use at least one of the following in your argument:

(i) the alternative form of the derivative $\lim_{x \to c} \frac{g(x) - g(c)}{x - c}$

(ii) one-sided limits

(iii) the graph of $g(x) = |x+2|$

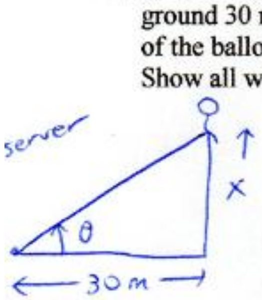


It is not differentiable at $c = -2$ because the tangent slope DNE.

OK

-0

12. **(11 points)** A balloon rises at the rate of 3 meters per second from a point on the ground 30 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 30 meters above the ground. Show all work.



$\tan\theta = \frac{x}{30}$

Diff wrt time

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{30}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{30}\frac{dx}{dt}\cos^2\theta$

When $x = 30$ m,

$\theta = 45^\circ$ so →

$\frac{d\theta}{dt} = \frac{1}{30}(3\ m/s)(\cos 45^\circ)^2$

$= \frac{1}{30}(3)\left(\frac{\sqrt{2}}{2}\right)^2$

$= \frac{1}{20}$ rad/sec

13. **(11 points)** Differential equations (DEs) are equations that naturally arise in physics and engineering applications. These equations involve an unknown function $f(t)$ and its derivatives. A common example from a spring mass system is the DE $y'' + y = 0$. Guess a solution $y = f(t)$ and test to see that it satisfies the DE. Many answers are possible.

$\cancel{e^{-x}}$

$\cancel{y' - e^{-x}}$

$\cancel{y'' = e}$

$y = \sin x$ ✓

$y' = \cos x$

$y'' = -\sin x$

$y = \sin x$ ✓ $\quad -\sin x + \sin x = 0$

Good