MTH 150
Exam 2
Spring 2014

Name: KEY

DIRECTIONS: This is a closed book, closed notes exam. No electronic devices are allowed (this means calculators, computers, cell phones, pagers, etc.). Be neat and show all work to receive full credit. Correct answers without the supporting evidence to back it up receive only partial credit. Good luck.

1. (a) **(4 points)** Given a function $y = f(x)$, state the **limit definition** of the derivative.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

(b) **(6 points)** Based on the definition from part (a), find $f'(x)$ for $f(x) = \frac{1}{x}$.

Using a shortcut rule doesn't count but you may *check* your answer via a shortcut.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(\frac{1}{x+\Delta x}\frac{x}{x} - \frac{1}{x}\frac{x+\Delta x}{x+\Delta x}\right)$$

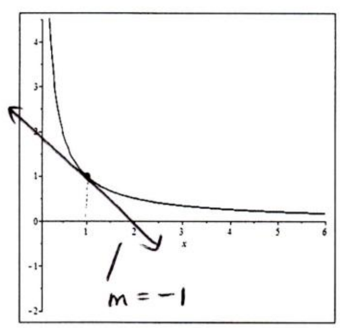
$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \frac{\cancel{x} - \cancel{x} - \Delta x}{x(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}}{\cancel{\Delta x} \cdot x(x+\Delta x)} =$$

$$\lim_{\Delta x \to 0} \frac{-1}{x(x+\underbrace{\cancel{\Delta x}}_{0})}$$

$$= \frac{-1}{x^2}$$

(c) **(4 points)** Calculate the value of $f'(1)$ by using part (b) above. Interpret this value with respect to the graph of $y = f(x)$ below.



$$f'(1) = \frac{-1}{(1)^2} = -1$$

slope of the tangent line to f at x = 1 (see diagram)

For problems 2-5, find the derivatives. Simplify to a reasonable point (e.g., eliminate complex fractions, negative exponents, etc.)

2. **(5 points)** $y = x^2 \ln x$

$$y' = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$$

$$= x + 2x \ln x$$

$$= x(1 + 2\ln x)$$

3. **(5 points)** $f(x) = \dfrac{1}{1 - \sin x}$

$$= (1 - \sin x)^{-1}$$

$$f'(x) = -1(1 - \sin x)^{-2}(-\cos x)$$

$$= \frac{\cos x}{(1 - \sin x)^2}$$

4. **(5 points)** $g(\theta) = \theta\sqrt{5 - \theta^2}$

$$= \theta\,(5 - \theta^2)^{1/2}$$

$$g'(\theta) = \theta\,\cancel{\tfrac{1}{2}}(5 - \theta^2)^{-1/2}(-\cancel{2}\theta) + \sqrt{5 - \theta^2} \cdot 1$$

$$= \frac{-\theta^2}{\sqrt{5 - \theta^2}} + \sqrt{5 - \theta^2} \cdot \frac{\sqrt{5 - \theta^2}}{\sqrt{5 - \theta^2}}$$

$$= \frac{-\theta^2 + 5 - \theta^2}{\sqrt{5 - \theta^2}}$$

$$= \frac{5 - 2\theta^2}{\sqrt{5 - \theta^2}}$$

5. **(5 points)** $h(x) = \tan^3(5x)$

$$= (\tan(5x))^3$$

$$h'(x) = 3(\tan(5x))^2 \sec^2(5x) \cdot 5$$

$$= 15 \tan^2 5x \sec^2 5x$$

Problems 6-12 are either short answer or multiple choice; no work is necessary. Circle the appropriate letter for the multiple choice items.

6. **(5 points)** Fill in the blank: Suppose that (a,b) lies on the graph of $y=f(x)$. If $f'(a)=5$, then $\left(f^{-1}\right)'(b)=$ $\underline{\quad \frac{1}{5} \quad}$.

7. **(5 points)** Which of the following is the derivative of $y=f(g(x))$?

A. $f'(g(x))$
B. $f'(g'(x))$
C. $f(g'(x))$
D. $f'(g(x))\cdot g'(x)$ ← Chain Rule
E. $f(g(x))\cdot g'(x)$

8. **(5 points)** Find the derivative: $f(x)=\arcsin(3x)$ → $f'(x) = \dfrac{3}{\sqrt{1-(3x)^2}}$

A. $\dfrac{1}{\sqrt{1-9x^2}}$
B. $\dfrac{3}{\sqrt{1-9x^2}}$
C. $\dfrac{18x}{\sqrt{1-9x^2}}$
D. $\dfrac{3}{\sqrt{1-3x^2}}$
E. $3\arccos(3x)$

9. **(5 points)** Given $y=\sin(xy)$, use implicit differentiation to find $\dfrac{dy}{dx}$.

A. $\dfrac{y\cos(xy)}{1-x\cos(xy)}$
B. $\dfrac{y\cos(xy)}{1-\cos(xy)}$
C. $\dfrac{y}{1-x}$
D. $\dfrac{y\cos(xy)}{y-x\cos(xy)}$
E. $\dfrac{1-y\cos(xy)}{x}$

$y' = \cos xy \cdot (xy' + y)$
$y' = xy'\cos xy + y\cos xy$
$y' - xy'\cos xy = y\cos xy$
$y'(1 - x\cos xy) = y\cos xy$
Divide

10. **(5 points)** Find the equation of the line tangent to the graph of $f(x)=e^{3x}-4x+2$ at the point $x=0$.

A. $y=-x+2$
B. $y=-4x+2$
C. $y=-3x+3$
D. $y=-x+3$
E. $y=-4x+8$

$f(0) = e^0 + 2 = 3$
point $(0,3)$

$f'(x) = 3e^{3x} - 4$
$f'(0) = 3e^0 - 4$
$= -1$ — slope

11. **(5 points)** Find the second derivative $\left(y'' \text{ or } \dfrac{d^2y}{dx^2}\right)$ of $y = \sec x$.

A. $\sec^3 x \tan x$

B. $\sec x \tan x$

C. $\sec x$

D. $\sec^2 x(\sec x + \tan x)$

E. $\sec x(\sec^2 x + \tan^2 x)$

$y' = \sec x \tan x$

$y'' = \sec x \cdot \sec^2 x$

$+ \tan x \cdot \sec x \tan x$

12. **(5 points)** $\dfrac{d}{dx}\left[e^{\sqrt{x}}\right] =$

A. $e^{\sqrt{x}}$

B. $\dfrac{1}{2}e^{\sqrt{x}} \cdot \ln x$

C. $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

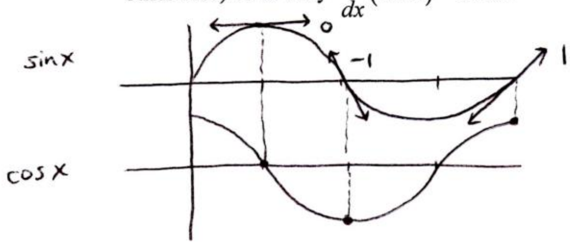
D. $\sqrt{x}\, e^{\sqrt{x}-1}$

E. None of these

$e^{\sqrt{x}} \cdot (\sqrt{x})'$

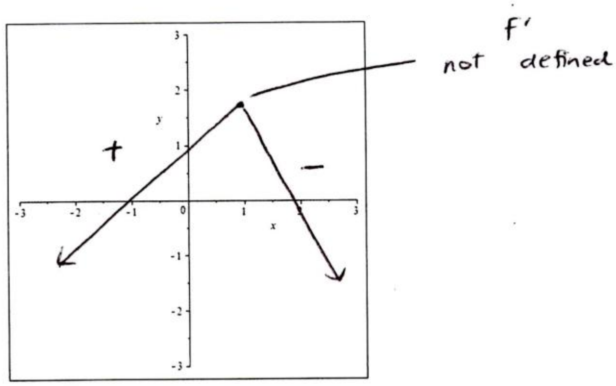
$= e^{\sqrt{x}} \, \frac{1}{2} x^{-1/2}$

13. **(5 points)** Give a convincing explanation (conceptual, graphical, analytical or otherwise) as to why $\dfrac{d}{dx}(\sin x) = \cos x$.



Whatever the slope of the TL is to the sine graph (0, -1, 1 are shown), this corresponds exactly to the value of the cosine graph at that value of x.

14. **(5 points)** Given the window below, sketch a function whose derivative starts positive, becomes undefined, and finishes negative. Use the space provided.



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DIRECTIONS: THERE ARE FOUR PROBLEMS ON THIS PART OF THE TEST; DO ANY THREE. Clearly indicate the one you are skipping. Calculators are permitted. However, answers based solely on calculator results are unacceptable. You must still show all work to receive full credit. Good luck.

15. A machine is causing a particle to move along the x-axis so that its position at time t is given by $x(t) = (t-4)^2$, where t is in seconds.

(a) **(3 points)** What is the particle's velocity at $t = 2$?

$v(t) = x'(t)$
$= 2(t-4)$

$v(2) = 2(2-4) = -4$ units/sec

it is moving to the left

(b) **(4 points)** The machine stops suddenly at $t = 3$ releasing the particle. As the particle continues, where will it be 5 seconds after the machine stops? Explain your thinking.

data when the machine stops
$x(3) = 1$
$v(3) = -2$

It is located at $x = 1$ & traveling at 2 units/sec to the left

Thus, after 5 seconds, the particle will be at

$x = \underbrace{1}_{\text{current location}} \underbrace{-2}_{\text{velocity}}(\underbrace{5}_{\text{time}}) = \boxed{-9}$

This will be its location

16. **(7 points)** Use Newton's method $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ to approximate the zero of the function $f(x) = x^3 + 2x + 1$. Use $x_0 = -1$ as an initial guess and find x_1, x_2, and x_3. Show the work for the computation of x_1; otherwise let the calculator do the work.

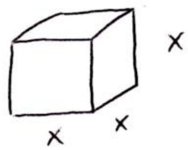
$f(x) = x^3 + 2x + 1$
$f'(x) = 3x^2 + 2$

$$x_1 = -1 - \frac{(-1)^3 + 2(-1) + 1}{3(-1)^2 + 2}$$

$= -0.6$

$x_2 \approx -0.465$
$x_3 \approx -0.453$

17. **(7 points)** The edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the volume changing when each edge is 1 cm?



$\frac{dx}{dt} = 3 \text{ cm/s}$ (given)

$\frac{dV}{dt} = ?$ when $x = 1$ cm

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$= 3(1)^2(3) = \boxed{9 \text{ cm}^3/\text{sec}}$

18. **(7 points)** Find the derivative of $y = x^x$.

$y = x^x$

$\ln y = \ln x^x$

$\ln y = x \cdot \ln x$ — diff implicitly

$\frac{y'}{y} = x \cdot \frac{1}{x} + \ln x \cdot 1$

$= 1 + \ln x$

$y' = y(1 + \ln x)$

$= \boxed{x^x(1 + \ln x)}$