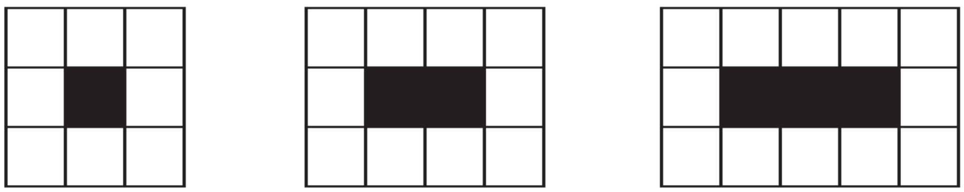
MATH 126
Problem Solving

Tiling a Patio

Alfredo Gomez is designing patios. Each patio has a rectangular garden area in the center. Alfredo uses black tiles to represent the soil of the garden. Around each garden, he designs a border of white tiles. The pictures shown below show the three smallest patios that he can design with black tiles for the garden and white tiles for the border.



Patio 1 **Patio 2** **Patio 3**

a. Draw patio 4 and patio 5. How many white tiles are in patio 4? Patio 5?

b. Make some observations about the patios that could help you describe larger patios.

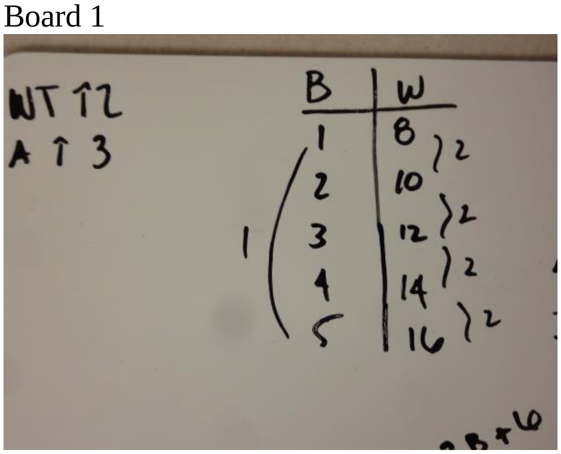
c. Describe a method for finding the total number of white tiles needed for patio 50 (without constructing it).

d. Write a rule that could be used to determine the number of white tiles needed for any patio. Explain how your rule relates to the visual representation of the patio.

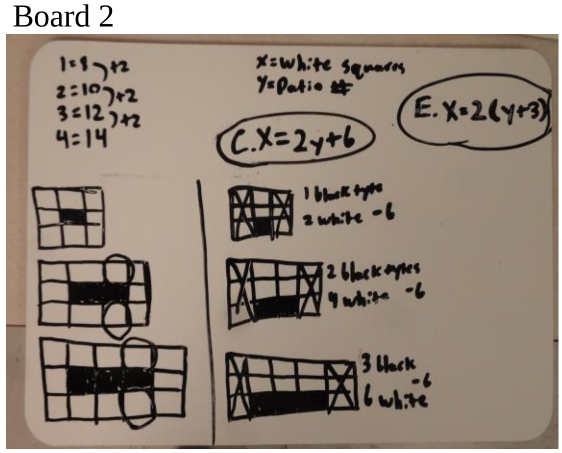
e. Write a different rule that could be used to determine the number of white tiles needed for any patio. Explain how your rule relates to the visual representation of the patio.

(Adapted from Cuevas and Yeatts [2005, pp. 18–22].)

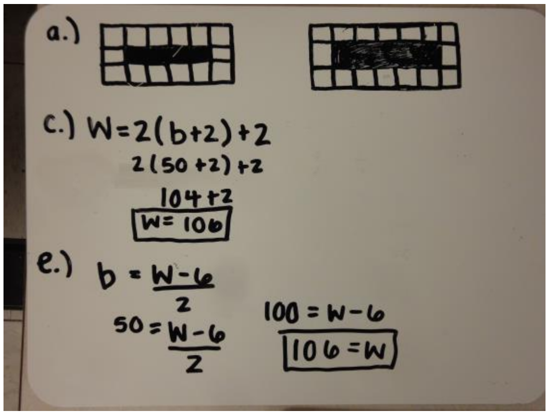
Board 1



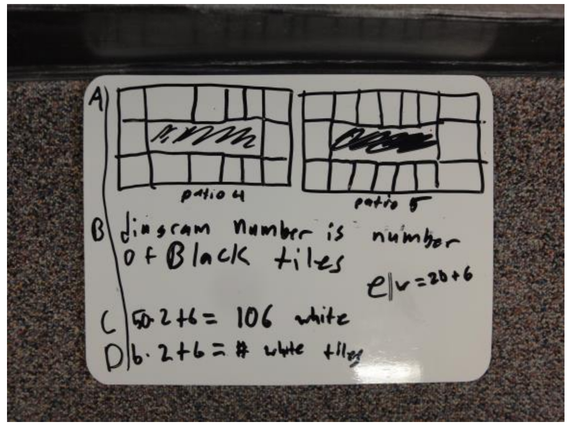
Board 2



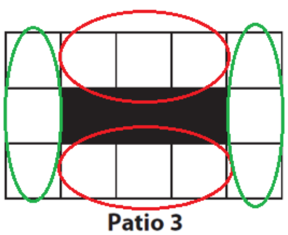
Board 3



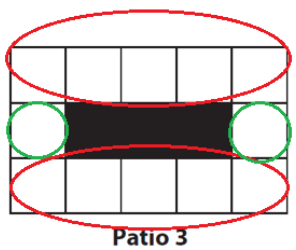
Board 4



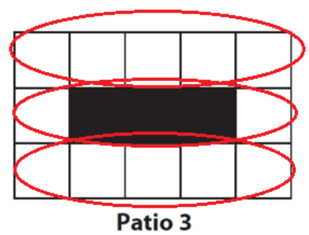
You can see on Board 1 and Board 2 that the first-order difference is constant (it's 2) so the relationship here is **not** quadratic; it will be linear. Based on the work here, we find that $w = 2b + 6$, where *w* stands for the number of white tiles and *b* stands for the number of black tiles (which is the same as the patio number). Here are a few different ways to get this formula. I'll use Patio 3 as a model.



Approach 1



Approach 2



Approach 3

Approach 1: This was most common. In red, you can see that you have the same number of white tiles as black tiles both above and below the black tiles. This gives $2b$. Then you can see the tiles circled in green (3 to the left, 3 to the right). Notice that for any patio number, it's always 3 and 3. Thus, you get 6 more white tiles. So $w = 2b + 6$. A similar approach was used in Board 2.

Approach 2: Look at one part circled in red. This is the same as the number of black tiles but with two extra—so, $b + 2$. However, there are two such sections so the parts circled in red show $2(b+2)$ white tiles. Then we need to get the remaining 2 white tiles (circled in green). Thus, we get $w = 2(b+2) + 2$. Notice this simplifies to $w = 2b + 6$. See Board 3.

Approach 3: Similar to Approach 2, you can just count **all** of the tiles first. This will give you $3(b+2)$. However, if you want only the white tiles, you now need to subtract the black tiles. Thus, subtract *b*. This gives $w = 3(b+2) - b$. Again, show that this is equivalent to $w = 2b + 6$.