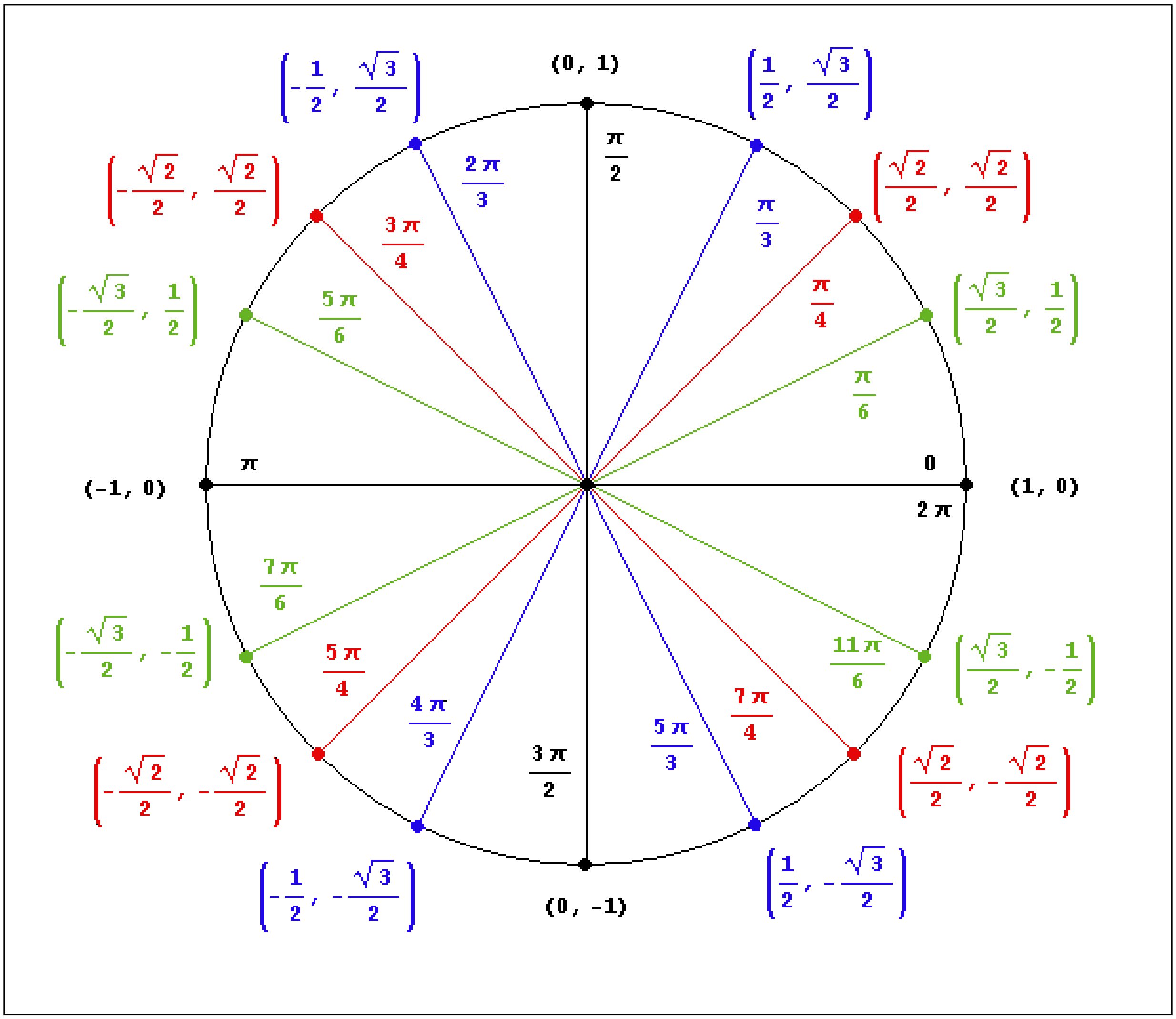
Trigonometry Review

I. The Unit Circle

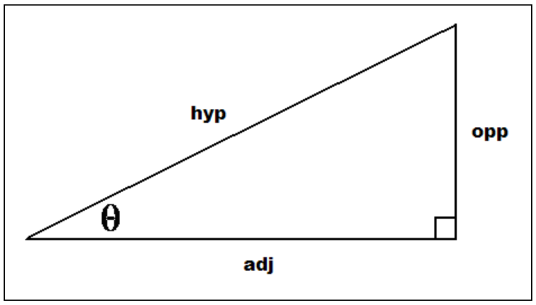


The above image came from http://members.aol.com/williamgunther/math/ref/unitcircle.gif.

θ is the angle measured counterclockwise from the positive *x*-axis. By definition, the value of the *x*-coordinate is the cosine function and the value of the *y*-coordinate is the sine function. In short, one can write $(x, y) = (\cos\theta, \sin\theta)$. For example, looking at the diagram above, $\cos\frac{\pi}{3} = \frac{1}{2}$. Also, $\sin\frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$. Try writing down several others. Notice that once the values in Quadrant I are determined, the others can be found quickly using the symmetry of the circle.

Note: Since the circle has radius 1, the *x*- and *y*-coordinates never exceed 1 in magnitude. Another way of saying this is $\boxed{-1 \le \sin\theta \le 1}$ and $\boxed{-1 \le \cos\theta \le 1}$.

II. The Six Trigonometric Functions



$$\sin\theta = \frac{\text{opp}}{\text{hyp}},\quad \cos\theta = \frac{\text{adj}}{\text{hyp}},\quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

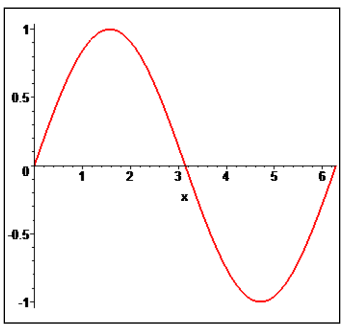
$$\csc\theta = \frac{\text{hyp}}{\text{opp}},\quad \sec\theta = \frac{\text{hyp}}{\text{adj}},\quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Note: Reciprocal and Quotient Identities follow directly from the definitions above.

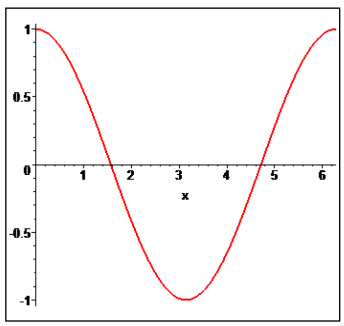
III. Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

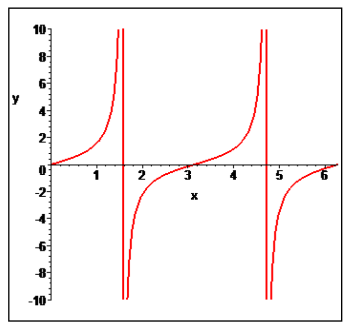
IV. Graphs of the Trigonometric Functions (on the window $[0, 2\pi]$)



$f(x) = \sin x$



$f(x) = \cos x$



$f(x) = \tan x$

V. Inverses

As an example, the **inverse sine function** is defined by $y = \arcsin x$ or $y = \sin^{-1} x$ if and only if $\sin y = x$. In the simplest terms, inverse functions are just like ordinary functions except the roles of *x* and *y* are reversed. For example, since $\cos\frac{\pi}{3} = \frac{1}{2}$, we may write $\arccos\frac{1}{2} = \frac{\pi}{3}$ or $\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$. From this, it may appear that writing $\cos a = b$ implies that $\arccos b = a$. This is often the case but not always. Anyone remember why?