

Two Algebra Problems

MATHEMAGICS?

Try this with a partner...Ask him/her to think of any positive integer (but keep it fairly small so you can do computations in your head. They should NOT reveal their number to you! Tell them to...

1. Square it.
2. Add the result to the original number.
3. Divide by your original number.
4. Add, oh, how about 17.
5. Subtract your original number.
6. Divide by 6.

Now tell them: The number now in your head is 3!
How is this possible?

To begin, start with a concrete number such as 4. Below, you can see several iterations of this—starting with numbers 9, 5, 2, 10, and 7. After following the instructions, you get the number 3 every time (see the bottom row). Something is definitely going on here!

9	5	2	10	7
81	25	4	100	49
90	30	6	110	56
10	6	3	11	8
27	23	20	28	25
18	18	18	18	18
3	3	3	3	3

Further, you might notice that all the scenarios show an 18 before the 3 (look at the second line up from the bottom). One way to see why this happens is to generalize the starting number and call it x . Then carefully watch what happens to x as you go through the steps 1-6 in the problem:

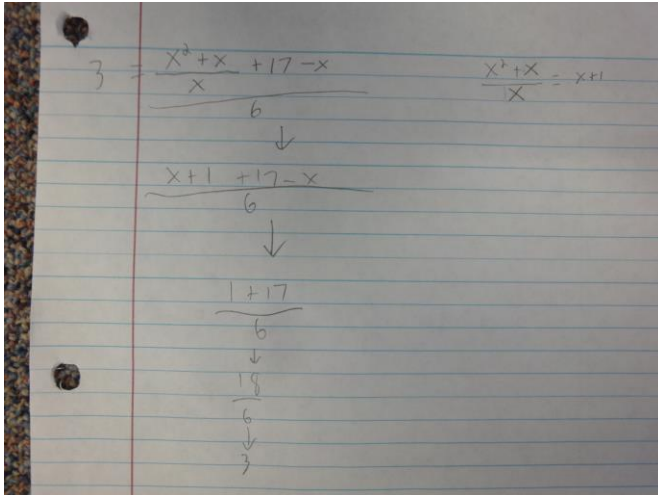
$$\begin{array}{l}
 x^2 \quad (1) \text{ square} \\
 x^2 + x \quad (2) \text{ + result to original} \\
 x + 1 \quad (3) \text{ } \div \text{ by original} \\
 x + 18 \quad (4) \text{ + 17} \\
 18 \quad (5) \text{ - original} \\
 (3) \quad (6) \text{ } \div \text{ 6} \\
 \downarrow \\
 \boxed{3}
 \end{array}$$

The six steps in detail.

$$\begin{array}{l}
 \sqrt{\quad} \quad \sqrt{\quad} \\
 \frac{x^2 + x}{x} = \frac{\cancel{x}(x+1)}{\cancel{x}} \\
 = x + 1
 \end{array}$$

The details for Step 3.

Some people chose to write one large expression for this process; it works! Study the details:



ALGEBRA GAFFE

Examine the work below...Can you identify the error?

Assume that $a = b$ and neither is zero. We will prove that $1 = 2$.

Begin with $a = b$. Then $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor both sides: $(a+b)(a-b) = b(a-b)$.

Cancel out the $a-b$ terms to get $a+b = b$.

Since $a = b$ by assumption, we have $b+b = b$ or $2b = b$. Dividing out b (which is not zero) gives $2 = 1$.

As we discussed in class, “canceling” the $a-b$ amounts to dividing by zero. Since division by zero is undefined, this explains the statement $2 = 1$.

